Vortex Formation behind an Object

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1. Vorticity Formation Mechanism

When two parallel uniform streams with different speeds meet at a plane, the fluid element on the boundary spins with is axis perpendicular to the stream. In an inviscid fluid, thickness of the boundary remains infinitesimal, which is called a "vortex sheet" (Batchelor, 1967). The vortex sheet is unstable, so that a small disturbance can create a wavy deformation, which rolls up to form a "vortex street" (Kelvin-Helmholtz instability), as illustrated in Figs. 1(a) and (b) (Drazin and Reid, 1981). In a viscous fluid, the vorticity created at the boundary diffuses into the fluid in the bulk, so that the thickness of the vortical region increases. Figure 1(c) shows an example, where the fluid flowing along the solid plane detaches and is carried in the downstream direction. The boundary becomes wavy and rolls up with increasing thickness. Such a region is called a "mixing layer" (see Fig. 1(c)).

2. Kármán Vortex Street

When a cylindrical body is placed in a viscous flow that is perpendicular to the axis of the cylinder, a double row of line vortices can be observed, which is called a Kármán vortex street (see Fig. 2), named after the fluid mechanist who analyzed its stability theoretically.

Strong vorticity regions are produced at the frontal surface of the cylinder, or any bluff bodies, whereas the fluid velocity in the back side of that body is very slow (called a "wake"), so that the line vortices similar to the ones shown in Fig. 1 and the ones with opposite sense of vorticity are formed on both sides of the wake and are released from the solid boundary under certain conditions. In the case of a circular cylinder of diameter d placed in a uniform flow of velocity U, the above-mentioned conditions are given in terms of the Reynolds number Re, defined by

$$Re = \frac{\rho U d}{\eta},$$

where ρ and η are, respectively, the density and viscosity of the fluid. Experiments show that

(a) $Re \lesssim 4$: no eddies are created behind the cylinder.

(b) $4 \lesssim Re \lesssim 40$: steady twin eddies are created behind the cylinder. Fluid that flows around the cylinder close to it moves away from it before reaching the rear point of symmetry. Fluid behind the cylinder circulates continuously, so that the steady attached eddies (twin eddies) are created behind the cylinder (see Fig. 3(b)). These eddies become bigger with the increase of Re.

(c) $Re \gtrsim 40$: Kármán vortex street is formed behind the cylinder. Symmetry of the twin-eddies is broken and the flow becomes unsteady, i.e., one side of them grows larger (Fig. 3(c)) and is shed into the fluid from the cylinder, followed by the other side similarly. The latter process is repeated periodically to form the vortices of the street (Fig. 3(c′)).

(d) The upper bound of Re where Kármán vortex street is observed is not clear, but a quasi-double row of vortices accompanied by turbulence is recognized above $Re \gtrsim 1000$.

The double row of vortex street was theoretically analyzed by by T. von Kármán (1911). Under the assumption of periodically placed vortex filaments in an inviscid fluid, he showed the stability condition $h/l = 0.281$, where l and h are, respectively, the distance between the two vortex filaments in the stream-wise direction and the distance between the two rows of street. Experiments show $h/l = 0.2 \sim 0.4$, which is in reasonable agreement. The real fluid, however, has a viscosity, which diffuses the size and magnitude of the vortical regions as well as the limited number of effective "vortex filaments" due to the energy dissipation. In fact the far-right vortex rings in Fig. 2 look elongated, which suggests that the magnitude of the vorticity is considerably reduced and that the dye used for flow visualization is simply convected.

3. Kármán Vortex Street in Nature and Engineering

Kármán vortex street, or that believed to be of the same character as the former, is reported in many geophysical flows (Lugt, 1983). Such an example can be seen in the cloud pattern behind an isolated high mountain or an island, where large-scale strong and steady wind, such as the prevailing westerly winds, passes through the object to form the vortex street.

Because of the periodic shedding of the Kármán vortex, which is accompanied by the lower pressure at the center of each vortex, a characteristic sound wave is generated. The periodicity of shedding is characterized by the Strouhal

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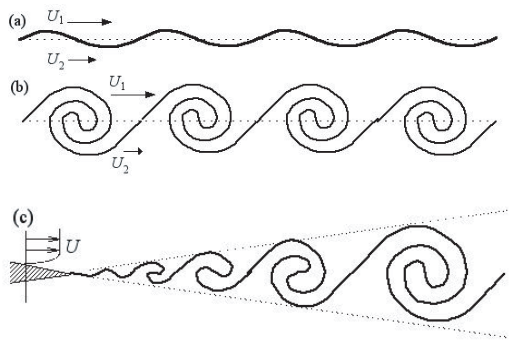

Fig. 1. (a), (b) Instability of a vortex sheet (Kelvin-Helmholtz instability). (c) Mixing layer.

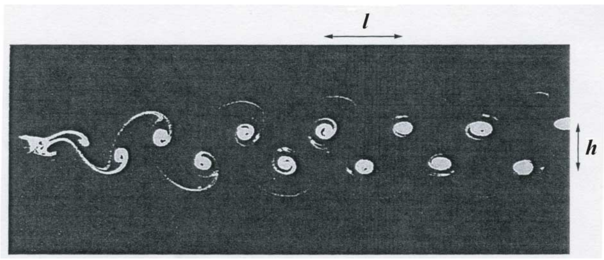

Fig. 2. Kármán vortex street at $Re = 105$ (van Dyke, 1982).

number St defined by

$$St = fd/U,$$

where f is the frequency of vortex shedding and d is the diameter of the cylinder. The value of St is experimentally found to be almost constant $St = 0.2 \sim 0.21$ for quite a wide range of Re (=500 $\sim 3.7 \times 10^5$) in an incompressible fluid. In the air, fluid properties ρ, μ are kept almost constant under the flow met in daily life, so that the frequency of the sound from a given cylinder can be a measure of the flow velocity. Some velocimetry using this relation is developed.

Each time when one of the vortex of the Kármán vortex street is released from the boundary of an object, this object experiences forces both in the stream-wise and sideways. Thus, if the object is not held firmly, that body starts an oscillation (flow-induced vibration), which may, associated with the forced oscillation combined with its structure, cause a collapse of bridge ("Tacoma Narrow Bridge", 1941), or a damage of cooling system of the nuclear plant ("Monju", 1995), etc.

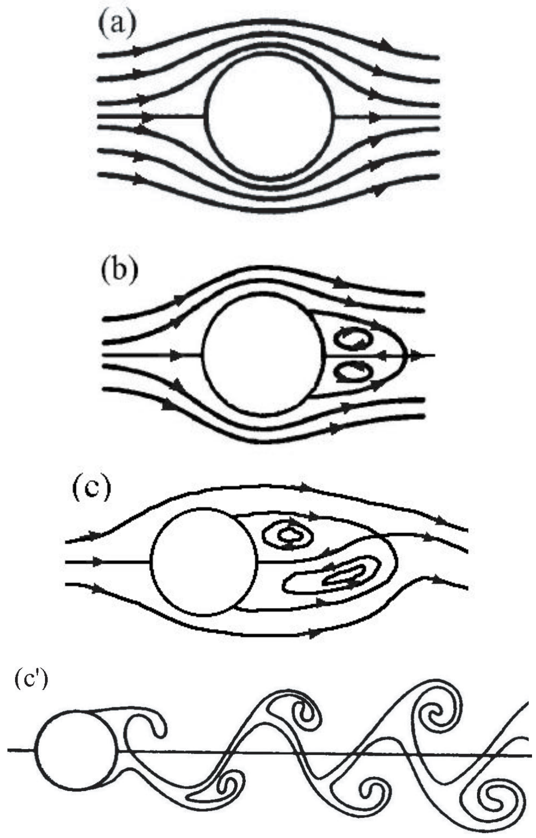

Fig. 3. (a) Low Reynolds number flow, where the upstream-downstream symmetry is maintained. (b) Steady twin eddies are formed behind the cylinder. (c) Symmetry and steadiness of the twin eddies no longer hold, and the eddies are shed into the fluid alternately. (c′) Fully developed double row of vortex street (Kármán vortex street).

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