

Analysis of Sufficient Density of Alternative Fuel Stations Using Rectilinear Distance

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This paper develops a model for determining sufficient density of alternative fuel stations. The service level is represented as the probability that the vehicle can make the repeated round trip between randomly selected origin and destination. Distance is measured as the rectilinear distance. The density of stations required to achieve a certain level of service is obtained for three cases: fuel is available at both origin and destination, fuel is available at either origin or destination, and fuel is available at neither origin nor destination. The result demonstrates how the vehicle range, the trip length, and the refueling availability at origin and destination affect the sufficient density of stations.

Key words: Location, Flow Demand, Vehicle Range, Trip Length, Round Trip

1. Introduction

Alternative fuel vehicles powered by electricity, hydrogen, and biofuels have been promoted because of environmental, geopolitical, and financial concerns. The transition from gasoline engine vehicles to alternative fuel vehicles, however, would be difficult. One of the most significant barriers to the transition is the scarcity of refueling stations (Greene, 1996).

Several approaches have been proposed to calculate the sufficient number of alternative fuel stations. Melaina (2003) developed three approaches for estimating the number of hydrogen stations based on the number of existing gasoline stations, metropolitan land areas, and lengths of major roads. Melaina and Bremson (2008) estimated the number of stations required to provide a sufficient level of coverage to all major urban areas. Nicholas *et al.* (2004) developed a GIS model for siting hydrogen stations and examined the effect of the number of stations on the average driving time to the nearest station. Nicholas and Ogden (2006) studied the regional variation in the number of stations needed to achieve a travel time target. Honma and Kurita (2008) obtained the optimal number of hydrogen stations that minimizes the sum of operation and transportation costs. Bersani *et al.* (2009) formulated a model for selecting gasoline stations to be converted to hydrogen stations.

Despite a large number of works concerning the sufficient number of stations, few studies have considered flow demand. Most of the previous studies assumed that drivers use their nearest station from their home. Refueling stations are, however, typical flow demand facilities in that demand for service can be expressed as flow (Hodgson, 1981, 1990; Berman *et al.*, 1992; Zeng *et al.*, 2010). In fact, drivers usually refuel their vehicles on pre-planned trips from origin to

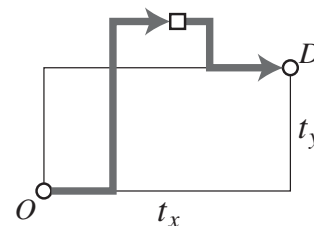


Fig. 1. Deviation to a station.

destination. This type of refueling behavior should be taken into account when discussing the sufficient number of stations.

In this paper, we develop a model for determining sufficient density of alternative fuel stations. To incorporate flow demand, the service level is represented as the probability that the vehicle can make the repeated round trip between randomly selected origin and destination. Although Miyagawa (2013a) proposed a similar model based on the Euclidean distance, we use the rectilinear distance instead. The reasons for using the rectilinear distance are as follows. First, the rectilinear distance is more suitable for cities with a grid road network (Love and Morris, 1979; Brimberg *et al.*, 2007; Griffith *et al.*, 2012). Second, the model with the rectilinear distance is more analytically tractable than that with the Euclidean distance.

An efficient location of alternative fuel stations has also been addressed. Kuby and Lim (2005) formulated the flow refueling location model (FRLM) for optimally locating refueling facilities. The FRLM locates p facilities to maximize the total flow volume that can be refueled. Kuby *et al.* (2009) applied the FRLM to the location of hydrogen stations in Florida. Lim and Kuby (2010) presented three heuristic algorithms for the FRLM. Capar and Kuby (2012) and Capar *et al.* (2013) proposed efficient formulations of

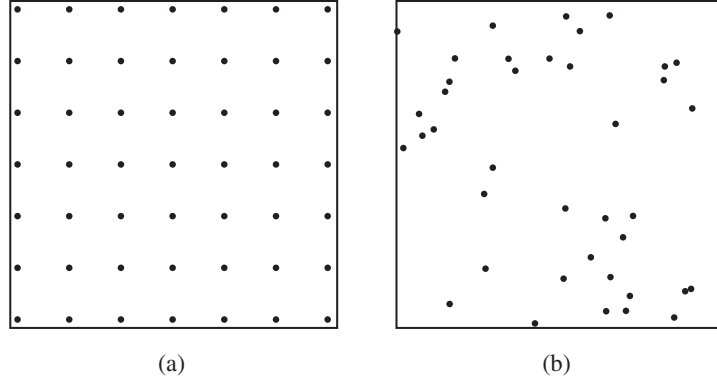


Fig. 2. Regular and random patterns: (a) grid; (b) random.

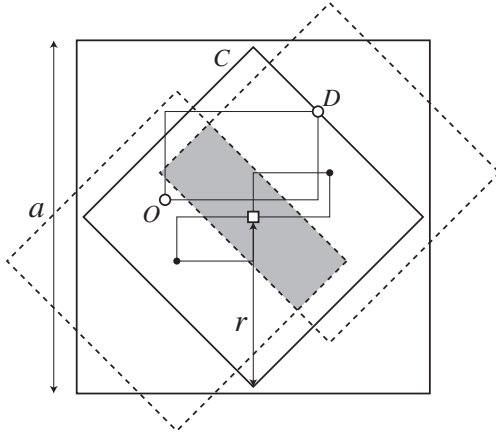


Fig. 3. Calculation of the probability for the grid pattern.

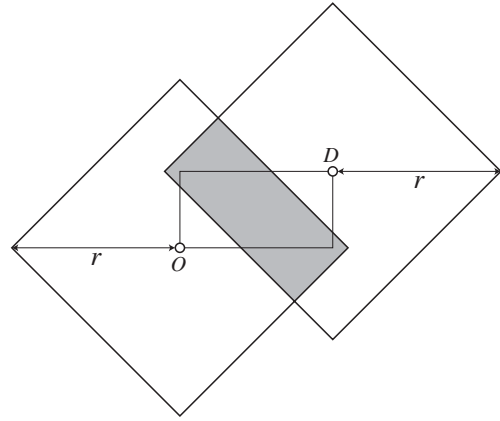


Fig. 4. Calculation of the probability for the random pattern.

the FRLM that make it possible to solve large problems. The FRLM was extended by Kuby and Lim (2007) to add candidate sites along network arcs, Upchurch *et al.* (2009) to include the capacity of refueling facilities, and Kim and Kuby (2012) to allow drivers to deviate from their shortest paths. Upchurch and Kuby (2010) compared the point-based p -median model and the flow-based FRLM. Wang and Lin (2009) presented a set-covering model to minimize the cost of refueling stations. In these location models, the number of stations to be located is an input. Our model will thus supplement further location models of stations.

The remainder of this paper is organized as follows. The next section develops a model for determining the sufficient density of alternative fuel stations. The following sections provide the sufficient density for three cases: fuel is available at both origin and destination, fuel is available at either origin or destination, and fuel is available at neither origin nor destination. The final section presents concluding remarks.

2. Model

Consider trips using alternative fuel vehicles. Let r be the vehicle range—the maximum distance that the vehicle with full tank of fuel can drive. Origins and destinations are selected at random within a study region. This assumption can yield analytical expressions for the probability of making the round trip. The analytical expressions provide fun-

damental relationships between variables, thereby serving as a basis for empirical analysis with actual travel demand. Let t be the trip length between origin and destination. Distance is measured as the rectilinear distance. The rectilinear distance between two points (x_1, y_1) and (x_2, y_2) is defined as $|x_1 - x_2| + |y_1 - y_2|$. Let t_x and t_y be the horizontal and vertical distances between origin and destination, respectively. The trip length is then $t = t_x + t_y$. Without loss of generality, we assume $t_x \geq t_y$. The set of the shortest paths between origin and destination is expressed as the rectangle with side lengths t_x and t_y , as shown in Fig. 1, which we call the shortest path rectangle.

Let $P(t)$ be the probability that the vehicle can make the repeated round trip between randomly selected origin and destination. Drivers are assumed to deviate from their shortest paths to refuel their vehicles, as shown in Fig. 1. Refueling is allowed only once for each one-way trip. Although only one refueling may not be enough for longer trips, we use this assumption for the following reasons. First, short distance trips are more frequent and thus more important than long distance trips with multiple refueling. Second, if multiple refueling is allowed for short distance trips, the sufficient density of stations decreases but the inconvenience of drivers increases. Finally, long distance trips should be addressed in discrete network models rather than continuous models, because long distance drivers usually use major highways, as discussed by Honma and Tori-

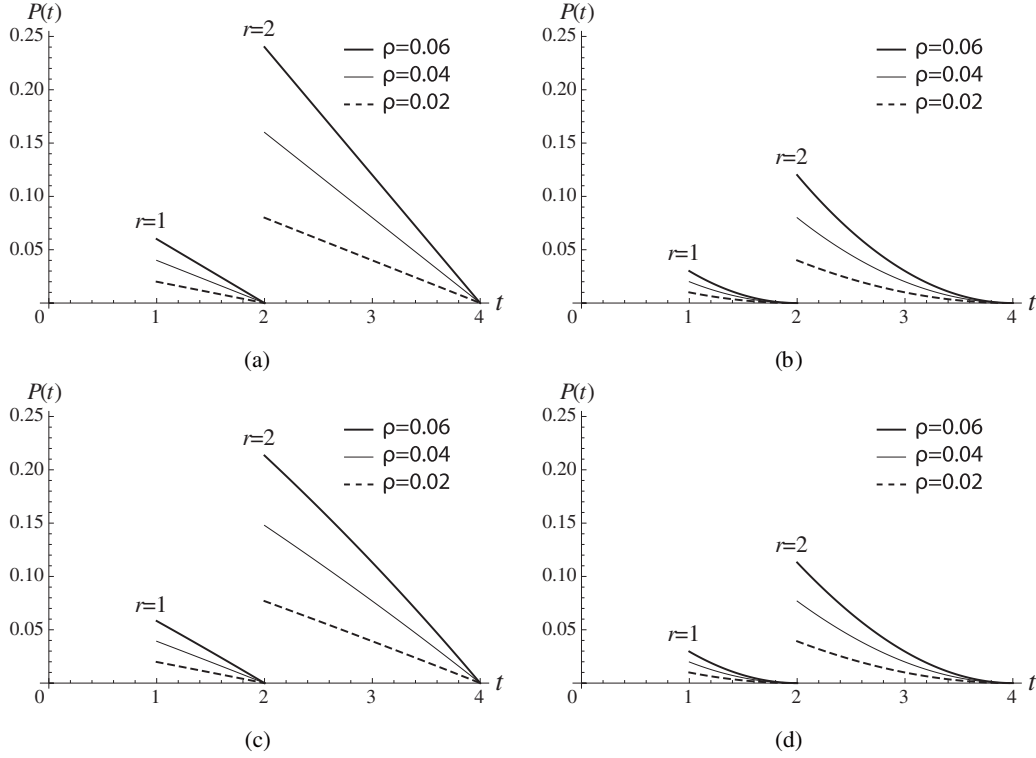


Fig. 5. Probability that the vehicle can make the repeated round trip: (a) grid ($t_x = t_y = t/2$); (b) grid ($t_x = t, t_y = 0$); (c) random ($t_x = t_y = t/2$); (d) random ($t_x = t, t_y = 0$).

Table 1. Density of stations required to achieve $P(t) \geq \alpha$.

α	Range of 1					Range of 2				
	Trip length t ($t_x = t_y = t/2$)					Trip length t ($t_x = t_y = t/2$)				
	0.75	1.00	1.25	1.50	1.75	1.50	2.00	2.50	3.00	3.50
0.2	0	0	0.30	0.45	0.89	0	0	0.07	0.11	0.22
0.4	0	0	0.68	1.02	2.04	0	0	0.17	0.26	0.51
0.6	0	0	1.22	1.83	3.67	0	0	0.31	0.46	0.92
0.8	0	0	2.15	3.22	6.44	0	0	0.54	0.80	1.61
α	Trip length t ($t_x = t, t_y = 0$)					Trip length t ($t_x = t, t_y = 0$)				
	0.75	1.00	1.25	1.50	1.75	1.50	2.00	2.50	3.00	3.50
	0.75	1.00	1.25	1.50	1.75	1.50	2.00	2.50	3.00	3.50
0.2	0	0	0.79	1.79	7.14	0	0	0.20	0.45	1.79
0.4	0	0	1.82	4.09	16.35	0	0	0.45	1.02	4.09
0.6	0	0	3.26	7.33	29.32	0	0	0.81	1.83	7.33
0.8	0	0	5.72	12.88	51.50	0	0	1.43	3.22	12.88

umi (2014).

Refueling stations are represented as points of regular and random patterns on a continuous plane, as shown in Fig. 2. Since actual patterns of stations can be regarded as intermediate between regular and random, the theoretical results of these extremes will give an insight into empirical studies on actual patterns. In fact, regular and random patterns have been used in location analysis (Larson and Odoni, 1981; O’Kelly and Murray, 2004; Miyagawa, 2013b, 2014). The regular and random patterns are assumed to be unbounded. This assumption enables us to examine the refueling availability without taking into account the edge effect.

3. Fuel is Available at Both Origin and Destination

First, we assume that fuel is available at both origin O and destination D . Then, the vehicle can start at O with full tank of fuel. If $t \leq r$, the vehicle can reach D without refueling, fill up at D , and return to O . If $t > 2r$, the vehicle cannot reach D , because more than one refueling is needed. Hence, we focus on the case where $r < t \leq 2r$. If $r < t \leq 2r$, the vehicle can make the round trip if both O and D are within the distance r of a station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , fill up again at D , turn round, fill up again at that same station, and return to O .

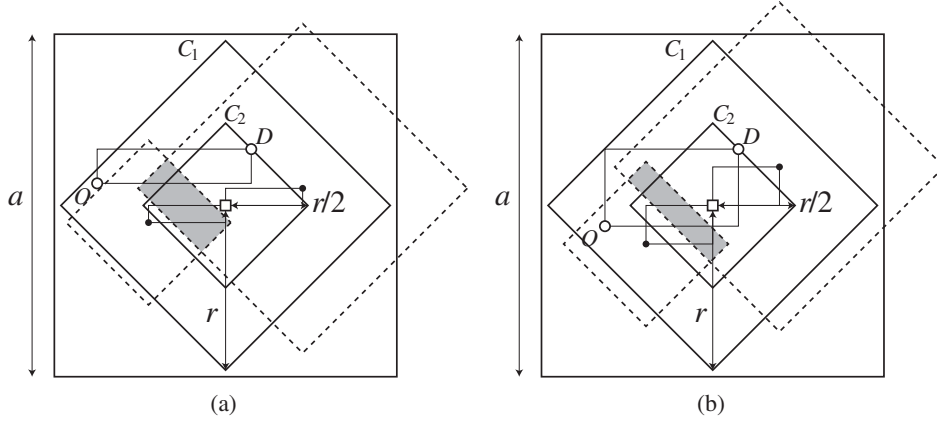


Fig. 6. Calculation of the probability for the grid pattern: (a) $t_y < r/2 \leq t_x - t_y$; (b) otherwise.

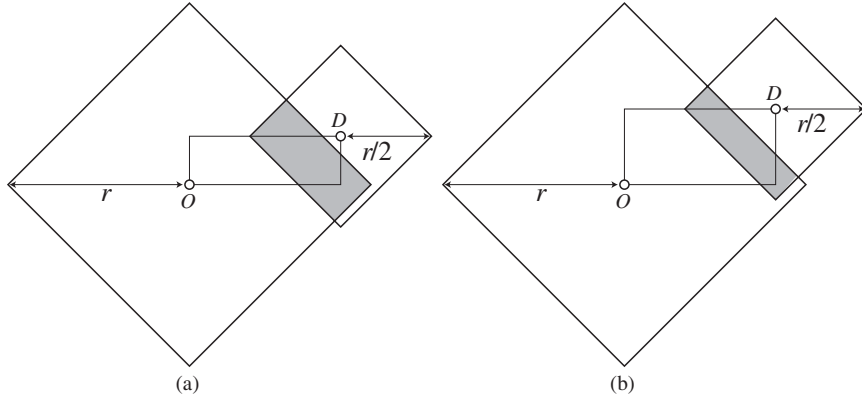


Fig. 7. Calculation of the probability for the random pattern: (a) $t_y < r/2 \leq t_x - t_y$; (b) otherwise.

3.1 Grid pattern

Suppose that stations are regularly distributed on a square grid with spacing a . The density of stations ρ , that is, the number of stations per unit area is then expressed as $\rho = 1/a^2$. The probability $P(t)$ can be calculated by considering only one station. The study region is then confined to the region where a station is the nearest, which is the square centered at the station with side length a , as depicted in Fig. 3.

To make the round trip, both O and D must be in the diamond C —a square rotated at angle 45° , centered at the station with radius r . Recall that a diamond gives the set of points within a given rectilinear distance from its center (Krause, 1987). If both O and D are in the diamond C , the center of the shortest path rectangle is in the shaded region in Fig. 3. This region is the intersection of the two diamonds which are obtained by moving the diamond C by $t_x/2$ to the right and $t_y/2$ to the upward and by $t_x/2$ to the left and $t_y/2$ to the downward. The intersection is assumed to be entirely within the square, i.e., $2r - t_x \leq a$, and the other case is left for future work. $P(t)$ is then the probability that the center of the shortest path rectangle lies inside the intersection of the two diamonds.

Since origins and destinations are selected at random, the center of the shortest path rectangle is uniformly distributed over the square. $P(t)$ is then given by the ratio of the area of the intersection to that of the square. The area of the

intersection of the two diamonds is

$$S = \frac{1}{2}(2r - t_x - t_y)(2r - t_x + t_y). \quad (1)$$

$P(t)$ is then

$$P(t) = \frac{S}{a^2} \quad (2)$$

$$= \frac{\rho}{2}(2r - t_x - t_y)(2r - t_x + t_y). \quad (3)$$

$P(t)$ is shown in Figs. 5a and b. $P(t)$ decreases with the trip length t , and increases with the density of stations ρ and the vehicle range r . Note that $P(t)$ for $t_x = t_y = t/2$ is greater than that for $t_x = t, t_y = 0$, even though the trip length is the same. That is, $P(t)$ varies according to the relative position of origin and destination as well as the trip length.

3.2 Random pattern

Suppose that stations are uniformly and randomly distributed. To make the round trip, both O and D must be within the distance r of a station. This means that the station must be in the intersection of the two diamonds centered at O and D with radius r , as depicted in Fig. 4. $P(t)$ is then the probability that the intersection of the two diamonds contains at least one station. The probability that a region of area S contains exactly x stations, denoted by $P(x, S)$, is given by the Poisson distribution as

$$P(x, S) = \frac{(\rho S)^x}{x!} \exp(-\rho S), \quad (4)$$

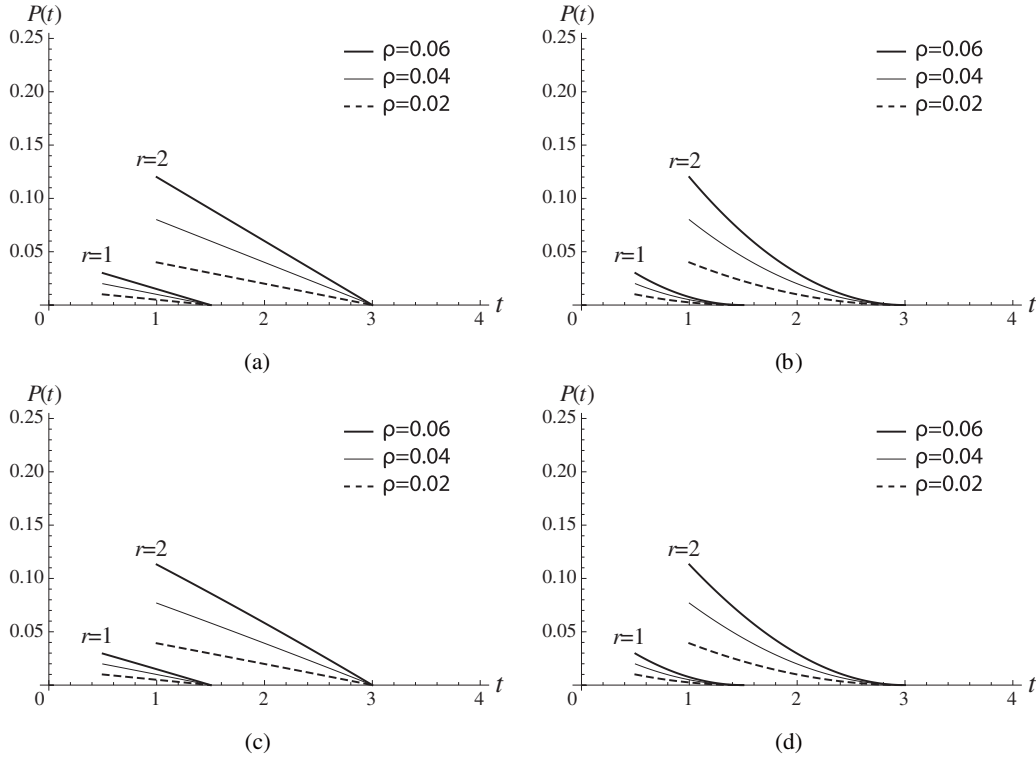


Fig. 8. Probability that the vehicle can make the repeated round trip: (a) grid ($t_x = t_y = t/2$); (b) grid ($t_x = t, t_y = 0$); (c) random ($t_x = t_y = t/2$); (d) random ($t_x = t, t_y = 0$).

Table 2. Density of stations required to achieve $P(t) \geq \alpha$.

α	Range of 1					Range of 2				
	Trip length t ($t_x = t_y = t/2$)					Trip length t ($t_x = t_y = t/2$)				
	0.50	0.75	1.00	1.25	1.50	1.00	1.50	2.00	2.50	3.00
0.2	0	0.60	0.80	1.79	–	0	0.15	0.22	0.45	–
0.4	0	1.36	2.04	4.09	–	0	0.34	0.51	1.02	–
0.6	0	2.44	3.67	7.33	–	0	0.61	0.92	1.83	–
0.8	0	4.29	6.44	12.88	–	0	1.07	1.61	3.22	–
α	Trip length t ($t_x = t, t_y = 0$)					Trip length t ($t_x = t, t_y = 0$)				
	0.50	0.75	1.00	1.25	1.50	1.00	1.50	2.00	2.50	3.00
	0.50	0.75	1.00	1.25	1.50	1.00	1.50	2.00	2.50	3.00
0.2	0	0.79	1.79	7.14	–	0	0.20	0.45	1.79	–
0.4	0	1.82	4.09	16.35	–	0	0.45	1.02	4.09	–
0.6	0	3.26	7.33	29.32	–	0	0.81	1.83	7.33	–
0.8	0	5.72	12.88	51.50	–	0	1.43	3.22	12.88	–

“–” means the round trip is impossible.

where ρ is the density of stations (Clark and Evans, 1954). From Eq. (1),

$$P(t) = 1 - P(0, S) \quad (5)$$

$$= 1 - \exp \left\{ -\frac{\rho}{2} (2r - t_x - t_y)(2r - t_x + t_y) \right\}. \quad (6)$$

$P(t)$ is shown in Figs. 5c and d. Note that $P(t)$ for the random pattern is slightly smaller than that for the grid pattern.

3.3 Density of stations

Using $P(t)$ for the grid and random patterns, we can calculate the density of stations required to achieve a specified level of service. Table 1 shows the density of stations required to achieve $P(t) \geq \alpha$ for the random pattern. Recall

that, if the trip length is shorter than the vehicle range, no station is needed. As the trip length becomes longer or the target probability increases, more stations are required. The target service level should be determined according to the traffic condition in the study region. For example, if long distance trips are dominant, we should use a large value for both t and α .

4. Fuel is Available at Either Origin or Destination

Next, we assume that fuel is available at either origin O or destination D . Without loss of generality, we assume that fuel is available at only O . Since the round trip is considered, the vehicle is required to reach D with at least half a tank remaining. If $t \leq r/2$, the vehicle can start at

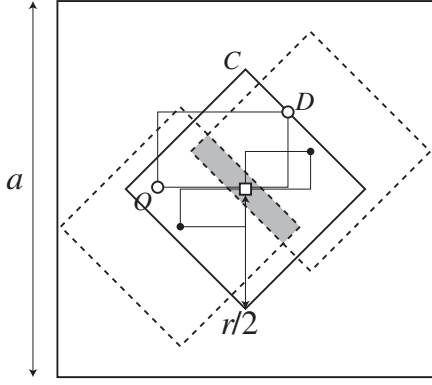


Fig. 9. Calculation of the probability for the grid pattern.

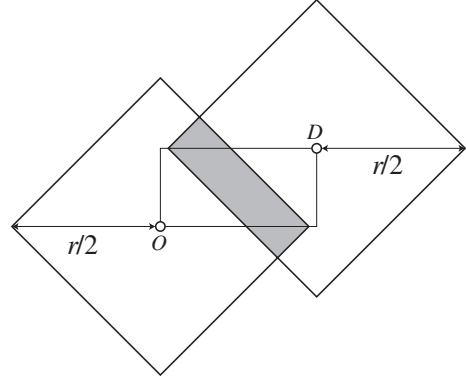


Fig. 10. Calculation of the probability for the random pattern.

O with full tank of fuel, reach D , and return to O without running out of fuel. If $t > 3r/2$, the vehicle cannot make the round trip without refueling more than once. Hence, we focus on the case where $r/2 < t \leq 3r/2$. If $r/2 < t \leq 3r/2$, the vehicle can make the round trip if O is within the distance r of a station and D is within the distance $r/2$ of the station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , turn round, fill up again at that same station, and return to O .

4.1 Grid pattern

To make the round trip, O must be in the diamond C_1 centered at the station with radius r and D must be in the diamond C_2 centered at the station with radius $r/2$, as depicted in Fig. 6. This means that the center of the shortest path rectangle must be in the shaded region in Fig. 6. This region is the intersection of the two diamonds which are obtained by moving the diamond C_1 by $t_x/2$ to the right and $t_y/2$ to the upward and moving the diamond C_2 by $t_x/2$ to the left and $t_y/2$ to the downward. $P(t)$ is then the probability that the center of the shortest path rectangle lies inside the intersection of the two diamonds.

The area of the intersection of the two diamonds is

$$S = \begin{cases} \frac{1}{2} \left(\frac{3}{2}r - t_x - t_y \right) \left(\frac{3}{2}r - t_x + t_y \right), & t_y < \frac{r}{2} \leq t_x - t_y, \\ \frac{r}{2} \left(\frac{3}{2}r - t_x - t_y \right), & \text{otherwise.} \end{cases} \quad (7)$$

From Eq. (2),

$$P(t) = \begin{cases} \frac{\rho}{2} \left(\frac{3}{2}r - t_x - t_y \right) \left(\frac{3}{2}r - t_x + t_y \right), & t_y < \frac{r}{2} \leq t_x - t_y, \\ \frac{\rho r}{2} \left(\frac{3}{2}r - t_x - t_y \right), & \text{otherwise.} \end{cases} \quad (8)$$

$P(t)$ is shown in Figs. 8a and b. It can be seen that $P(t)$ is smaller than that of the previous case (see Fig. 5).

4.2 Random pattern

To make the round trip, O must be within the distance r of a station and D must be within the distance $r/2$ of

the station. This means that the station must be in the intersection of the diamond centered at O with radius r and the diamond centered at D with radius $r/2$, as depicted in Fig. 7. $P(t)$ is then the probability that the intersection of the two diamonds contains at least one station. From Eqs. (5) and (7),

$$P(t) = \begin{cases} 1 - \exp \left\{ -\frac{\rho}{2} \left(\frac{3}{2}r - t_x - t_y \right) \left(\frac{3}{2}r - t_x + t_y \right) \right\}, & t_y < \frac{r}{2} \leq t_x - t_y, \\ 1 - \exp \left\{ -\frac{\rho r}{2} \left(\frac{3}{2}r - t_x - t_y \right) \right\}, & \text{otherwise.} \end{cases} \quad (9)$$

$P(t)$ is shown in Figs. 8c and d.

4.3 Density of stations

Table 2 shows the density of stations required to achieve $P(t) \geq \alpha$ for the random pattern. Recall that long distance trips are impossible, because multiple refueling is needed. More stations are required than the previous case to achieve the same level of service (see Table 1).

5. Fuel is Available at Neither Origin nor Destination

Finally, we assume that fuel is available at neither origin O nor destination D . We also assume that the vehicle starts at O with half a tank of fuel and reaches D with at least half a tank remaining, as suggested by Kuby and Lim (2005). This assumption ensures that the vehicle can make the repeated round trip. If $t > r$, the vehicle cannot make the round trip without refueling more than once. Hence, we focus on the case where $t \leq r$. If $t \leq r$, the vehicle can complete the round trip with at least half a tank remaining if both O and D are within the distance $r/2$ of a station (Miyagawa 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , turn round, fill up again at that same station, and return to O .

5.1 Grid pattern

To make the round trip, both O and D must be in the diamond C centered at the station with radius $r/2$, as depicted in Fig. 9. This means that the center of the shortest path rectangle must be in the shaded region in Fig. 9. This region is the intersection of the two diamonds which are

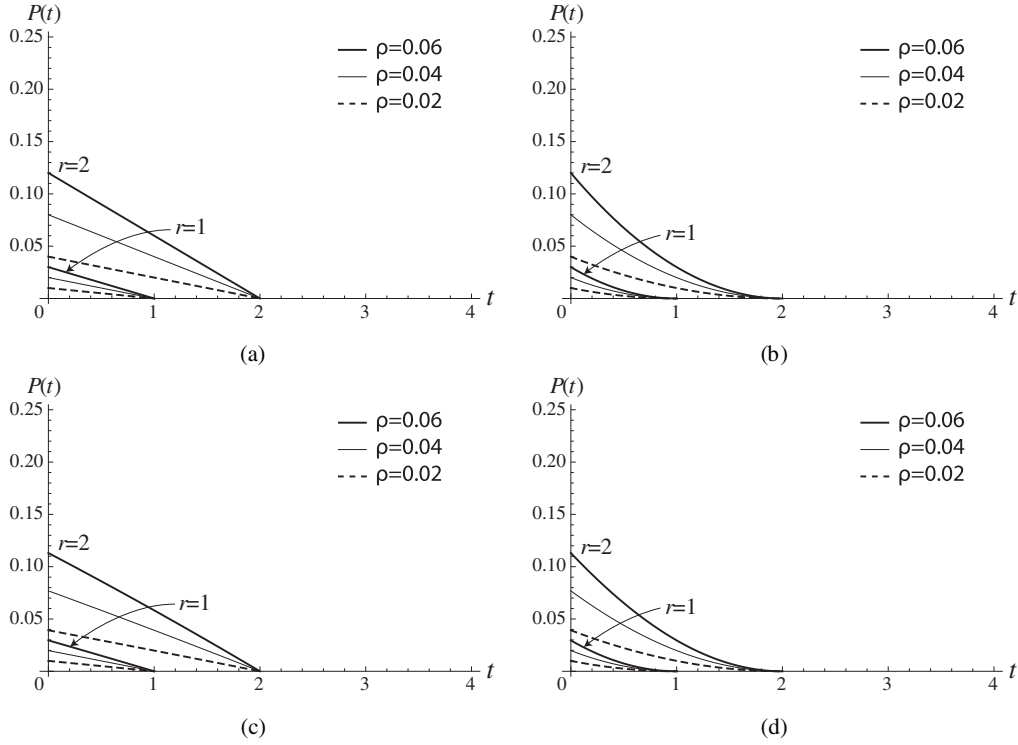


Fig. 11. Probability that the vehicle can make the repeated round trip: (a) grid ($t_x = t_y = t/2$); (b) gird ($t_x = t, t_y = 0$); (c) random ($t_x = t_y = t/2$); (d) random ($t_x = t, t_y = 0$).

Table 3. Density of stations required to achieve $P(t) \geq \alpha$.

α	Range of 1					Range of 2				
	Trip length t ($t_x = t_y = t/2$)					Trip length t ($t_x = t_y = t/2$)				
	0.25	0.50	0.75	1.00	1.25	0.50	1.00	1.50	2.00	2.50
0.2	0.60	0.89	1.79	—	—	0.15	0.22	0.45	—	—
0.4	1.36	2.04	4.09	—	—	0.34	0.51	1.02	—	—
0.6	2.44	3.67	7.33	—	—	0.61	0.92	1.83	—	—
0.8	4.29	6.44	12.88	—	—	1.07	1.61	3.22	—	—
α	Trip length t ($t_x = t, t_y = 0$)					Trip length t ($t_x = t, t_y = 0$)				
	0.25	0.50	0.75	1.00	1.25	0.50	1.00	1.50	2.00	2.50
0.2	0.79	1.79	7.14	—	—	0.20	0.45	1.79	—	—
0.4	1.82	4.09	16.35	—	—	0.45	1.02	4.09	—	—
0.6	3.26	7.33	29.32	—	—	0.81	1.83	7.33	—	—
0.8	5.72	12.88	51.50	—	—	1.43	3.22	12.88	—	—

“—” means the round trip is impossible.

obtained by moving the diamond C by $t_x/2$ to the right and $t_y/2$ to the upward and by $t_x/2$ to the left and $t_y/2$ to the downward. $P(t)$ is then the probability that the center of the shortest path rectangle lies inside the intersection of the two diamonds.

The area of the intersection of the two diamonds is

$$S = \frac{1}{2}(r - t_x - t_y)(r - t_x + t_y). \quad (10)$$

From Eq. (2),

$$P(t) = \frac{\rho}{2}(r - t_x - t_y)(r - t_x + t_y). \quad (11)$$

$P(t)$ is shown in Figs. 11a and b. In this case, even short distance trips are not always possible.

5.2 Random pattern

To make the round trip, both O and D must be within the distance $r/2$ of a station. This means that the station must be in the intersection of the two diamonds centered at O and D with radius $r/2$, as depicted in Fig. 10. $P(t)$ is then the probability that the intersection of the two diamonds contains at least one station. From Eqs. (5) and (10),

$$P(t) = 1 - \exp\left\{-\frac{\rho}{2}(r - t_x - t_y)(r - t_x + t_y)\right\}. \quad (12)$$

$P(t)$ is shown in Figs. 11c and d.

5.3 Density of stations

Table 3 shows the density of stations required to achieve $P(t) \geq \alpha$ for the random pattern. As expected, more stations are required than the other two cases.

6. Conclusions

This paper has developed a model for determining sufficient density of alternative fuel stations. The model based on the rectilinear distance is suitable for regions where the rectilinear distance is a good approximation for the actual travel distance.

The probability of making the round trip has been obtained for regular and random patterns of stations. The probability provides a rough estimate for the service level of actual patterns. The analytical expressions for the probability demonstrate how the density of stations, the vehicle range, the trip length, and the refueling availability at origin and destination affect the service level of alternative fuel stations. Note that finding these relationships by using discrete network models requires computation of the number of origin-destination pairs that can make the round trip for various combinations of the parameters. The relationships help policy makers to estimate the number of stations required to achieve a certain level of service. The estimated number of stations can be used as an input in location models of alternative fuel stations. Comparing the effects on the service level is also useful to prioritize investments for the transition to alternative fuel vehicles.

The model assumes that origins and destinations are randomly distributed within a study region. Since travel demand generally depends on the trip length, introducing the trip length distribution is an important topic for future research.

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References

- Berman, O., Larson, R. and Fouska, N. (1992) Optimal location of discretionary service facilities, *Transportation Science*, **26**, 201–211.
- Bersani, C., Minciardi, R., Sacile, R. and Trasforini, E. (2009) Network planning of fuelling service stations in a near-term competitive scenario of the hydrogen economy, *Socio-Economic Planning Sciences*, **43**, 55–71.
- Brimberg, J., Walker, J. and Love, R. (2007) Estimation of travel distances with the weighted ℓ_p norm: Some empirical results, *Journal of Transport Geography*, **15**, 62–72.
- Capar, I. and Kuby, M. (2012) An efficient formulation of the flow refueling location model for alternative-fuel stations, *IIE Transactions*, **44**, 622–636.
- Capar, I., Kuby, M., Leon, V. and Tsai, Y.-J. (2013) An arc cover-path-cover formulation and strategic analysis of alternative-fuel station locations, *European Journal of Operational Research*, **227**, 142–151.
- Clark, P. and Evans, F. (1954) Distance to nearest neighbor as a measure of spatial relationships in populations, *Ecology*, **35**, 85–90.
- Greene, D. (1996) Survey evidence on the importance of fuel availability to the choice of alternative fuels and vehicles, *Energy Studies Review*, **8**, 215–231.
- Griffith, D., Vojnovic, I. and Messina, J. (2012) Distances in residential space: Implications from estimated metric functions for minimum path distances, *GIScience & Remote Sensing*, **49**, 1–30.
- Hodgson, M. (1981) The location of public facilities intermediate to the journey to work, *European Journal of Operational Research*, **6**, 199–204.
- Hodgson, M. (1990) A flow-capturing location-allocation model, *Geographical Analysis*, **22**, 270–279.
- Honma, Y. and Kurita, O. (2008) A mathematical model on the optimal number of hydrogen stations with respect to the diffusion of fuel cell vehicles, *Journal of the Operations Research Society of Japan*, **51**, 166–190.
- Honma, Y. and Toriumi, S. (2014) Model analysis of electric vehicle charging infrastructure development on highways—An approximation of the required scale of electric vehicle charging facilities—, *FORMA*, **29**, 41–50.
- Kim, J.-G. and Kuby, M. (2012) The deviation-flow refueling location model for optimizing a network of refueling stations, *International Journal of Hydrogen Energy*, **37**, 5406–5420.
- Krause, E. (1987) *Taxicab Geometry: An Adventure in Non-Euclidean Geometry*, Dover, New York.
- Kuby, M. and Lim, S. (2005) The flow-refueling location problem for alternative-fuel vehicles, *Socio-Economic Planning Sciences*, **39**, 125–145.
- Kuby, M. and Lim, S. (2007) Location of alternative-fuel stations using the flow-refueling location model and dispersion of candidate sites on arcs, *Networks and Spatial Economics*, **7**, 129–152.
- Kuby, M., Lines, L., Schultz, R., Xie, Z., Kim, J.-G. and Lim, S. (2009) Optimization of hydrogen stations in Florida using the flow-refueling location model, *International Journal of Hydrogen Energy*, **34**, 6045–6064.
- Larson, R. and Odoni, A. (1981) *Urban Operations Research*, Prentice Hall, Englewood Cliffs, NJ.
- Lim, S. and Kuby, M. (2010) Heuristic algorithms for siting alternative-fuel stations using the flow-refueling location model, *European Journal of Operational Research*, **204**, 51–61.
- Love, R. and Morris, J. (1979) Mathematical models of road travel distances, *Management Science*, **25**, 130–139.
- Melaina, M. (2003) Initiating hydrogen infrastructures: Preliminary analysis of a sufficient number of initial hydrogen stations in the US, *International Journal of Hydrogen Energy*, **28**, 743–755.
- Melaina, M. and Bremson, J. (2008) Refueling availability for alternative fuel vehicle markets: Sufficient urban station coverage, *Energy Policy*, **36**, 3233–3241.
- Miyagawa, M. (2013a) Density of alternative fuel stations and refueling availability, *International Journal of Hydrogen Energy*, **38**, 12438–12445.
- Miyagawa, M. (2013b) Distribution of the difference between distances to the first and the second nearest facilities, *Journal of the Operations Research Society of Japan*, **56**, 167–176.
- Miyagawa, M. (2014) Distribution of the sum of distances to the first and second nearest facilities, *Geographical Analysis*, **46**, 321–333.
- Nicholas, M. and Ogden, J. (2006) Detailed analysis of urban station siting for California hydrogen highway network, *Transportation Research Record*, **1983**, 121–128.
- Nicholas, M., Handy, S. and Sperling, D. (2004) Using geographic information systems to evaluate siting and networks of hydrogen stations, *Transportation Research Record*, **1880**, 126–134.
- O’Kelly, M. and Murray, A. (2004) A lattice covering model for evaluating existing service facilities, *Papers in Regional Science*, **83**, 565–580.
- Upchurch, C. and Kuby, M. (2010) Comparing the p -median and flow-refueling models for locating alternative-fuel stations, *Journal of Transport Geography*, **18**, 750–758.
- Upchurch, C., Kuby, M. and Lim, S. (2009) A model for location of capacitated alternative-fuel stations, *Geographical Analysis*, **41**, 85–106.
- Wang, Y.-W. and Lin, C.-C. (2009) Locating road-vehicle refueling stations, *Transportation Research Part E*, **45**, 821–829.
- Zeng, W., Castillo, I. and Hodgson, M. (2010) A generalized model for locating facilities on a network with flow-based demand, *Networks and Spatial Economics*, **10**, 579–611.