

Deviation Distance and Sufficient Density of Alternative Fuel Stations

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This paper presents a model for determining the sufficient density of alternative fuel stations. The model extends a previous model to incorporate both flow demand and the deviation distance, and provides a more appropriate framework for analyzing the density of stations. The service level is represented as the probability that the vehicle can make the repeated round trip between randomly selected origin and destination within a specified deviation distance. The density of stations required to achieve a certain level of service is obtained for three cases: fuel is available at both origin and destination, fuel is available at either origin or destination, and fuel is available at neither origin nor destination. The result shows how the deviation distance, the vehicle range, the trip length, and the refueling availability at origin and destination affect the sufficient density of stations.

Key words: Location, Flow Demand, Vehicle Range, Trip Length, Round Trip

1. Introduction

Alternative fuel vehicles powered by electricity, hydrogen, and biofuels have been promoted because of concerns about climate change and energy security. One of the most significant disadvantages of alternative fuel vehicles is the scarcity of refueling stations (Greene, 1996). Adequate availability of stations would accelerate the transition from gasoline engine vehicles to alternative fuel vehicles.

Several approaches have been proposed to calculate the sufficient number of alternative fuel stations. Melaina (2003) developed three approaches for estimating the number of hydrogen stations based on the number of existing gasoline stations, metropolitan land areas, and lengths of principal arterial roads. Melaina and Bremson (2008) estimated the number of stations required to provide a sufficient level of coverage to all major urban areas. Nicholas *et al.* (2004) presented a model for siting hydrogen stations and examined the effect of the number of stations on the average driving time from home or workplace to the nearest station. Nicholas and Ogden (2006) studied the regional variation in the number of stations needed to achieve a travel time target. Honma and Kurita (2008) obtained the optimal number of hydrogen stations that minimizes the sum of operation and transportation costs. Bersani *et al.* (2009) formulated a model for selecting gasoline stations to be converted to hydrogen stations.

Most of the studies reviewed above examined the distance to the nearest station, assuming that drivers use the nearest station from their home. Refueling stations are, however, typical flow demand facilities in that demand for service can be expressed as flow (Hodgson, 1990; Berman *et al.*, 1992; Zeng *et al.*, 2010). In fact, drivers usually refuel their vehicles on pre-planned trips from origin to destination.

Nicholas (2010) and Kelley and Kuby (2013) found that more drivers choose a station on their least deviation route than the station closest to home. The flow demand was introduced into the estimation of the sufficient number of stations by Miyagawa (2013a), who focused on whether the vehicle can make the round trip between origin and destination.

An overlooked by Miyagawa (2013a) but significant element is the deviation distance—the travel distance when drivers deviate from their pre-planned paths to visit a facility. The deviation distance has frequently been addressed in flow demand location models (Hodgson, 1981; Berman *et al.*, 1995; Berman, 1997; Tanaka and Furuta, 2012). Analytical expressions for the deviation distance were derived by Miyagawa (2010) for general flow demand facilities and Miyagawa (2013b) for alternative fuel stations. Since refueling demand generally decreases with the deviation distance to visit a station, the deviation distance should be considered when estimating the sufficient number of stations.

In this paper, we present a model for determining the sufficient density of alternative fuel stations required to achieve a certain level of service. To incorporate both flow demand and the deviation distance, we focus on whether the vehicle can make the round trip within a specified deviation distance. The service level is represented as the probability of making the round trip between randomly selected origin and destination within a deviation distance. The present model therefore extends the model by Miyagawa (2013a). We then examine how the deviation distance, the vehicle range, the trip length, and the refueling availability at origin and destination affect the sufficient density of stations.

Facility location models based on flow demand have been used to optimally locate alternative fuel stations. Kuby and Lim (2005) developed the flow refueling location model (FRLM), which locates p facilities to maximize the total flow volume that can be refueled. Kuby *et al.* (2009) ap-

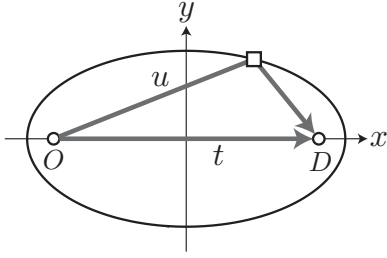


Fig. 1. Deviation distance to visit a station.

plied the FRLM to the location of hydrogen stations in Florida. Lim and Kuby (2010) presented three heuristic algorithms for the FRLM. Capar and Kuby (2012) and Capar *et al.* (2013) proposed efficient formulations of the FRLM to solve large problems. The FRLM was extended by Kuby and Lim (2007) to add candidate sites along network arcs, Upchurch *et al.* (2009) to include the capacity of refueling facilities, and Kim and Kuby (2012) to allow drivers to deviate from their shortest paths. Wang and Lin (2009) presented a set-covering model to minimize the cost of refueling stations. In these location models, the number of stations to be located is an input. Our model will thus supplement location models of alternative fuel stations.

The rest of this paper is organized as follows. The next section develops a model for determining the sufficient density of alternative fuel stations. The following sections provide the density of stations required to achieve a specified level of service for three cases of the refueling availability at origin and destination. The final section presents concluding remarks.

2. Model

Consider trips using alternative fuel vehicles. Let r be the vehicle range—the maximum distance that the vehicle with full tank of fuel can drive. Origins and destinations are selected at random within a study region. The random travel demand can be used as the first approximation for the actual travel demand and serves as a basis for further analysis with more realistic travel demand. For example, travel demand that depends on the trip length can be considered by incorporating the trip length distribution, as discussed by Miyagawa (2016).

Drivers are assumed to deviate from their shortest paths to refuel their vehicles. Let t be the trip length between origin O and destination D and u be the deviation distance to visit a station. The deviation distance is defined as the sum of the distances from O to the station and from the station to D . Distance is measured as the Euclidean distance. The region that a driver can cover within a deviation distance u forms an ellipse, the foci of which are at O and D . Recall that an ellipse is defined as the locus of points such that the sum of the distances to two fixed points (foci) remains constant. Set the coordinate system as shown in Fig. 1. The ellipse is then expressed as

$$\frac{4x^2}{u^2} + \frac{4y^2}{u^2 - t^2} = 1, \quad (1)$$

where $u \geq t$ (Miyagawa, 2013b). If the ellipse contains a

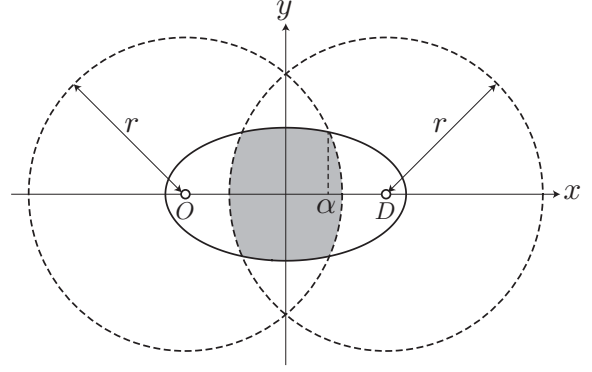
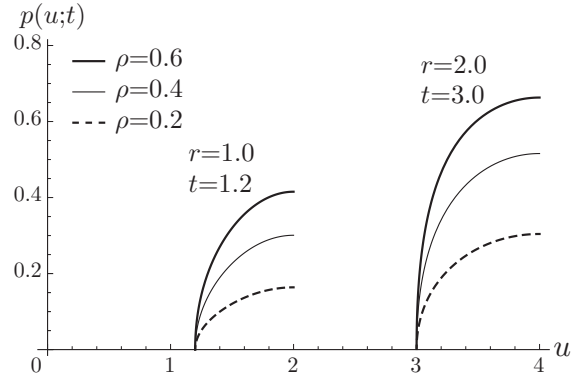


Fig. 2. Calculation of the probability.

Fig. 3. Probability of making the round trip within a deviation distance u .

station, the station is available within the deviation distance u .

Let $p(u; t)$ be the probability that the vehicle can make the repeated round trip between randomly selected origin and destination within a deviation distance u . Refueling is allowed only once for each one-way trip. Thus, we focus on short distance trips which need at most one refueling. Note that if multiple refueling is allowed for such short distance trips, the sufficient density of stations decreases, but the inconvenience of drivers increases. Since $p(u; t)$ depends on the refueling availability at origin and destination, three cases are considered: fuel is available at both origin and destination, fuel is available at either origin or destination, and fuel is available at neither origin nor destination. The first or second case can be applied to plug-in electric vehicles, whereas the third case can be applied to hydrogen and natural gas vehicles.

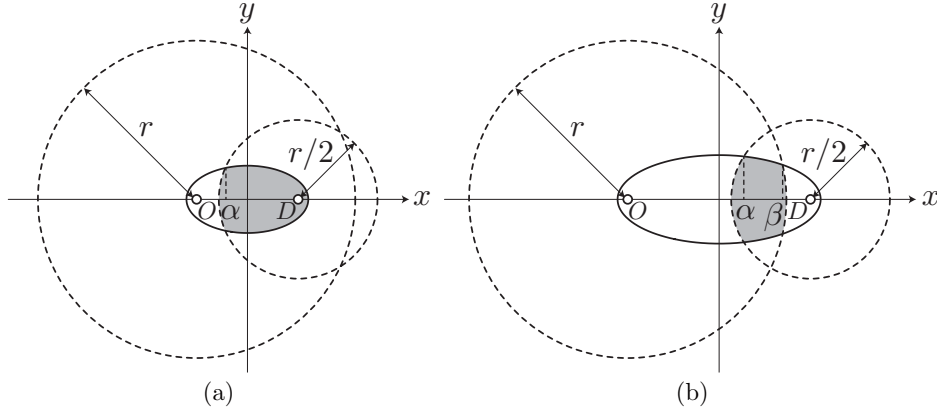
Refueling stations are assumed to be randomly distributed. This assumption is not entirely unrealistic because when stations are sparse in an early stage of the development, the pattern of stations makes little impact on the basic properties of the probability of making the round trip. By comparing the probabilities for grid and random patterns of stations, Miyagawa (2013a) demonstrated that the difference between them is relatively small.

3. Fuel is Available at Both Origin and Destination

First, we assume that fuel is available at both origin O and destination D . The vehicle can then start at O with

Table 1. Density of stations required to achieve $p(u; t) \geq \alpha$.

α	t u	1.2			1.4			1.6		
		1.3	1.4	1.5	1.5	1.6	1.7	1.7	1.8	1.9
0.2		0.61	0.43	0.35	0.73	0.53	0.45	1.05	0.79	0.71
0.4		1.39	0.97	0.80	1.68	1.21	1.02	2.40	1.82	1.62
0.6		2.49	1.75	1.43	3.02	2.18	1.84	4.31	3.26	2.90
0.8		4.38	3.07	2.51	5.30	3.83	3.23	7.57	5.72	5.09

Fig. 4. Calculation of the probability: (a) $t \leq u \leq 2r - t$; (b) $2r - t < u \leq 3r/2$.

full tank of fuel. If $t \leq r$, the vehicle can reach D without refueling and return to O . If $t > 2r$, the vehicle cannot reach D because more than one refueling is needed. Hence, we focus on the case where $r < t \leq 2r$. If $r < t \leq 2r$, the vehicle can make the round trip if both O and D are within the distance r of a station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , fill up again at D , turn round, fill up again at that same station, and return to O .

To refuel at a station and complete the round trip, the station must be in the intersection of the two circles centered at O and D with radius r . To visit the station within a deviation distance u , the station must also be in the ellipse (1). Thus, $p(u; t)$ is the probability that the intersection of the two circles and the ellipse contains at least one station, as shown in Fig. 2. The probability that a region of area S contains exactly x stations, denoted by $P(x, S)$, is given by the Poisson distribution as

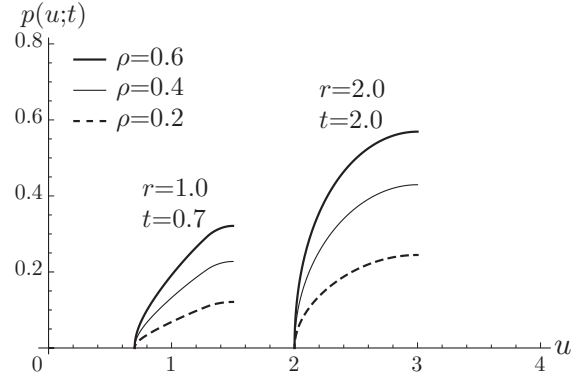
$$P(x, S) = \frac{(\rho S)^x}{x!} \exp(-\rho S), \quad (2)$$

where ρ is the density of stations (Clark and Evans, 1954). The area of the intersection is, if $t \leq u \leq 2r$,

$$S = \frac{2\sqrt{u^2 - t^2}}{u} \int_0^\alpha \sqrt{u^2 - 4x^2} dx + 4 \cdot \int_\alpha^{r-t/2} \sqrt{r^2 - \left(x + \frac{t}{2}\right)^2} dx, \quad (3)$$

where

$$\alpha = \frac{2ru - u^2}{2t}. \quad (4)$$

Fig. 5. Probability of making the round trip within a deviation distance u .

The probability $p(u; t)$ is obtained as

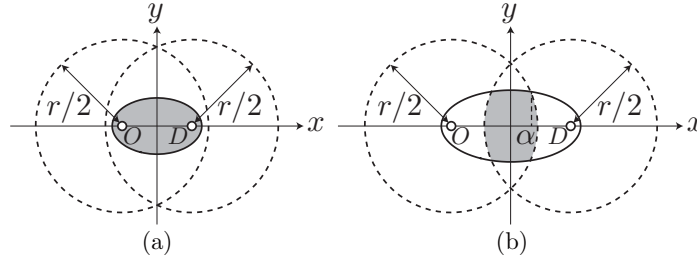
$$p(u; t) = 1 - P(0, S) = 1 - \exp(-\rho S). \quad (5)$$

Although the final form is not provided due to the limited space, the probability can be expressed in a closed form. The probability $p(u; t)$ is shown in Fig. 3. It can be seen that $p(u; t)$ increases with the deviation distance u and the density of stations ρ . Note that $p(t; t) = 0$ because the vehicle cannot make a deviation and that $p(2r; t)$ is identical with the result obtained by Miyagawa (2013a).

Using the probability $p(u; t)$, we can calculate the density of stations required to achieve a specified level of service. Table 1 shows the density of stations required to achieve $p(u; t) \geq \alpha$ for the vehicle range $r = 1$. The required density increases with the trip length t and the target probability α , and decreases with the deviation distance u that drivers can tolerate. The target service level should be

Table 2. Density of stations required to achieve $p(u; t) \geq \alpha$.

α	t u	0.7			0.9			1.1		
		0.8	0.9	1.0	1.0	1.1	1.2	1.2	1.3	1.4
0.2		1.28	0.83	0.63	1.20	0.79	0.63	1.41	1.03	0.90
0.4		2.93	1.90	1.44	2.75	1.81	1.44	3.22	2.37	2.07
0.6		5.26	3.41	2.59	4.93	3.25	2.58	5.77	4.25	3.71
0.8		9.24	5.99	4.54	8.66	5.70	4.53	10.14	7.46	6.52

Fig. 6. Calculation of the probability: (a) $t \leq u \leq r - t$; (b) $r - t < u \leq r$.

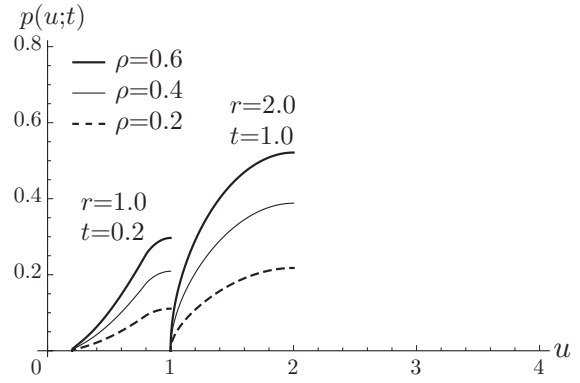
determined according to the traffic condition in the study region. If long distance trips are dominant, we should use a large value for both t and α . If drivers are reluctant to make a deviation to refuel their vehicles, the value for u should not be much greater than that for t .

4. Fuel is Available at Either Origin or Destination

Next, we assume that fuel is available at either origin O or destination D . Without loss of generality, we assume that fuel is available at only O . Since the round trip is considered, the vehicle is required to reach D with at least half a tank remaining. If $t \leq r/2$, the vehicle can make the round trip without refueling. If $t > 3r/2$, the vehicle cannot make the round trip without refueling more than once. Hence, we focus on the case where $r/2 < t \leq 3r/2$. If $r/2 < t \leq 3r/2$, the vehicle can make the round trip if O is within the distance r of a station and D is within the distance $r/2$ of the station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , turn round, fill up again at that same station, and return to O .

To refuel at a station and complete the round trip, the station must be in the intersection of the circle centered at O with radius r and the circle centered at D with radius $r/2$. To visit the station within a deviation distance u , the station must also be in the ellipse (1). Thus, $p(u; t)$ is the probability that the intersection of the two circles and the ellipse contains at least one station, as shown in Fig. 4. The area of the intersection is, if $t \leq u \leq 2r - t$,

$$S = \frac{\sqrt{u^2 - t^2}}{u} \int_{\alpha}^{u/2} \sqrt{u^2 - 4x^2} dx + 2 \int_{t/2 - r/2}^{\alpha} \sqrt{\left(\frac{r}{2}\right)^2 - \left(x - \frac{t}{2}\right)^2} dx, \quad (6)$$

Fig. 7. Probability of making the round trip within a deviation distance u .

and if $2r - t < u \leq 3r/2$,

$$S = \frac{\sqrt{u^2 - t^2}}{u} \int_{\alpha}^{\beta} \sqrt{u^2 - 4x^2} dx + 2 \int_{t/2 - r/2}^{\alpha} \sqrt{\left(\frac{r}{2}\right)^2 - \left(x - \frac{t}{2}\right)^2} dx + 2 \int_{\beta}^{r - t/2} \sqrt{r^2 - \left(x + \frac{t}{2}\right)^2} dx, \quad (7)$$

where

$$\alpha = \frac{u^2 - ru}{2t}, \quad \beta = \frac{2ru - u^2}{2t}. \quad (8)$$

The probability $p(u; t)$ is obtained from Eq. (5) and shown in Fig. 5. Note that $p(3r/2; t)$ is identical with the result obtained by Miyagawa (2013a).

Table 2 shows the density of stations required to achieve $p(u; t) \geq \alpha$ for $r = 1$. Observe that more stations are required than the previous case to achieve an even lower level of service.

Table 3. Density of stations required to achieve $p(u; t) \geq \alpha$.

α	t u	0.2			0.4			0.6		
		0.3	0.4	0.5	0.5	0.6	0.7	0.7	0.8	0.9
0.2		4.24	2.05	1.24	1.89	1.06	0.77	1.70	1.22	1.05
0.4		9.70	4.69	2.84	4.34	2.42	1.77	3.89	2.78	2.39
0.6		17.39	8.42	5.09	7.78	4.35	3.17	6.98	5.00	4.29
0.8		30.55	14.79	8.94	13.66	7.64	5.56	12.27	8.77	7.54

5. Fuel is Available at Neither Origin nor Destination

Finally, we assume that fuel is available at neither origin O nor destination D . We also assume that the vehicle starts at O with half a tank of fuel and reaches D with at least half a tank remaining, as suggested by Kuby and Lim (2005). This assumption ensures that the vehicle can make the repeated round trip. If $t > r$, the vehicle cannot make the round trip without refueling more than once. Hence, we focus on the case where $t \leq r$. If $t \leq r$, the vehicle can complete the round trip with at least half a tank remaining if both O and D are within the distance $r/2$ of a station (Miyagawa, 2013a). In fact, the vehicle can reach the station, fill up at the station, go to D , turn round, fill up again at that same station, and return to O .

To refuel at a station and complete the round trip, the station must be in the intersection of the two circles centered at O and D with radius $r/2$. To visit the station within a deviation distance u , the station must also be in the ellipse (1). Thus, $p(u; t)$ is the probability that the intersection of the two circles and the ellipse contains at least one station, as shown in Fig. 6. The area of the intersection is, if $t \leq u \leq r - t$,

$$S = \frac{\pi u}{4} \sqrt{u^2 - t^2}, \quad (9)$$

and if $r - t < u \leq r$,

$$S = \frac{2\sqrt{u^2 - t^2}}{u} \int_0^u \sqrt{u^2 - 4x^2} dx + 4 \int_{\alpha}^{r/2-t/2} \sqrt{\left(\frac{r}{2}\right)^2 - \left(x + \frac{t}{2}\right)^2} dx, \quad (10)$$

where

$$\alpha = \frac{ru - u^2}{2t}. \quad (11)$$

The probability $p(u; t)$ is obtained from Eq. (5) and shown in Fig. 7. Note that $p(r; t)$ is identical with the result obtained by Miyagawa (2013a).

Table 3 shows the density of stations required to achieve $p(u; t) \geq \alpha$ for $r = 1$. As expected, more stations are required than the other two cases.

6. Conclusions

This paper has extended a previous model for determining the sufficient density of alternative fuel stations to consider the deviation distance. The service level is represented as the probability that the vehicle can make the repeated

round trip between randomly selected origin and destination within a specified deviation distance. The model incorporates both flow demand and the deviation distance, and thus provides a more appropriate framework for analyzing the sufficient density of stations.

The analytical expressions for the probability demonstrate how the density of stations, the deviation distance, the vehicle range, the trip length, and the refueling availability at origin and destination affect the service level. Note that finding these relationships by using discrete network models requires computation of the number of origin-destination pairs that can make the round trip for various combinations of the parameters. The relationships enable us to estimate the number of stations required to achieve a certain level of service. The estimated number of stations can be used as an input in location models of alternative fuel stations. The relationships are also useful to evaluate the effect of policies to support infrastructure development.

The proposed model can be extended in future research. First, not only the deviation distance but also the distance from home to the nearest station should be considered. The proximity to home might be important for the purchase decision of alternative fuel vehicles. Second, estimating the number of vehicles refueled and the quantity of fuel needed at each station is necessary to determine the capacity of stations. The estimation involves more realistic travel demand such as spatial interaction models (Roy, 2010). Finally, the refueling time cannot be ignored particularly for electric vehicles.

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