

Survival Time of Princess Kaguya in an Air-Tight Bamboo Chamber

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Princess Kaguya is a heroine of a famous folk tale, as every Japanese knows. She was assumed to be confined in a bamboo cavity with cylindrical shape, and then fortuitously discovered by an elderly man in the forest. Here, we pose a question as to how long she could have survived in an enclosed space such as the bamboo chamber, which had no external oxygen supply at all. We demonstrate that the survival time should be determined by three geometric attributes regarding her body and the bamboo chamber. We also emphasize that this geometric problem shed light on an interesting scaling relation between biological quantities for living organisms.

Key words: Metabolic Rate, Respiration, Allometry, Biophysics

1. Introduction

Imagine being trapped in a closed room with no window, chimney, or hole in the wall to allow the flow of air. Would you survive for long or remain safe in that situation?

The most serious and imminent danger associated with entrapment in a sealed room is the inevitable development of oxygen deficiency disease. Oxygen is one of the key elements required for the sustenance of life, as experimentally confirmed in the 1770's by Joseph Priestley [1]. The author demonstrated that mice could survive in closed containers as long as they contained plants that emit oxygen through photosynthesis; otherwise, the mice could not survive without the plants. This may be true for all mammals including humans [2]. By incorporating oxygen into the body, humans create approximately 30 molecules of the energy source adenosine triphosphate (ATP) from one molecule of glucose [3]. Therefore, once all the oxygen in a sealed room is consumed by an entrapped individual, the body can no longer produce energy without which it would be difficult to survive. Here is the question: How long can a human being survive in a sealed room with no external oxygen supply?

This article provides a viable solution to the question posed. Our argument is based on a fictional mysterious girl “Princess Kaguya”, who is the heroine of the most famous folk tale in Japan [4]. In the tale, an old man working as bamboo cutter discovered a miniature girl inside a glowing bamboo shoot; see Fig. 1. Believing her to be a divine presence, he and his wife decided to raise her as their own child. When the girl came of age, she was conferred with the formal name “Princess Kaguya”, which incorporates the Japanese term Kaguya derived from the light and life she radiated. The legendary story of Princess Kaguya has been handed down through generations [5]; it was animated in 2013, released worldwide, and was finally nominated for a



Fig. 1. Cartoon of the situation where a girl was discovered by an old man. The folk tale states that the old man found a small girl sitting inside a bamboo chamber that shined like gold. He then took her home, named her Princess Kaguya, and nurtured her.

2015 Academy Award for a feature length animation film [6].

As mentioned above, Princess Kaguya was assumed to have been discovered inside the stalk of a bamboo plant (see Fig. 2), which was completely air-tight. This poses the question of how long Princess Kaguya had survived in the bamboo chamber. We will see below that solving this problem may facilitate understanding of the interplay between metabolism and respiration [8, 9]. In addition, this problem can be easily extended to situations on different scales, and is not limited to a “miniature girl situation”; it thus provides an intellectual exercise for considering the allometric relationship of living matter with respect to oxygen.



Fig. 2. Cross section of a bamboo culm segment. Reprinted from Ref. [7] with permission.

2. Assumed Condition

Discussion will be based on the hypotheses listed below.

- The space where Princess Kaguya was trapped had cylindrical shape and was a completely sealed room with no inflow and outflow of air.
- The height of the Princess was 9.10 cm [4] and did not change while she was trapped.
- The Princess remained sitting quietly in the space until her destined discovery by the old bamboo cutter.
- Her body produced energy only by breathing the oxygen inside the enclosed space.

3. Way of Thinking

3.1 Viable time of Princess Kaguya

Suppose that the inner volume of the bamboo chamber in which the Princess was trapped is V_{inn} [L]. The chamber was filled with air and initially, the oxygen concentration could be assumed to be 21%, similar to that of ambient air. For humans in general (including Princess Kaguya), the oxygen concentration of exhaled breath is 16%. Figure 3 provides a visual illustration; the outer large square boundary represents the inner chamber that encloses the flesh air with 21% in oxygen concentration at the initial state. The total air inside the boundary is divided into those confined in many small square domains, each of which represents the amount of air that Princess inhales and exhales by one breath. For simplicity, every exhaled breath does not diffuse but remain in the same position. * As a result, the oxygen concentration in a small domain (highlighted by gray) is displaced by 16% following each breath. Every single breath, Princess inhales flesh air in different small domains in order to intake a constant amount of oxygen. Eventually when she breaths half the volume of the total air in the chamber, a net oxygen concentration will be 18.5% at the final state (see the bottom of Fig. 3). This value is nearly the same as the lower limit under which oxygen deficiency occurs. Considering the volume V_{Pr} that Princess' body occupies in the chamber, $(V_{\text{inn}} - V_{\text{Pr}}) \times 0.50$ is the total volume of air that the Princess could breathe for survival.

Let Q_{air} be the amount of air that Princess intakes per

*If the diffusion of exhaled breath is taken into account, the net oxygen concentration in the inner chamber will decrease with time and Princess will breathe fast by actively moving diaphragm and muscles needed breathing. This may cause the increase in the consumed energy per hour and thus result in a decline of Princess' survival time compared with that obtained under the diffusion-free condition.

Table 1. Metabolic rates for selected activities [8].

Activity	Metabolic rate [kcal/(m ² · h)]
Sleeping	35
Sitting upright	50
Standing	60
Walking	140
Running	600

unit time. From the above argument, the viable time T for Princess reads

$$T = \frac{(V_{\text{inn}} - V_{\text{Pr}}) \times 0.50}{Q_{\text{air}}}. \quad (1)$$

In the following discussion, we first evaluate the Princess-related parameters: Q_{air} and V_{Pr} in Sections 3.2 and 3.3, and then address the estimation of the bamboo-related parameter V_{inn} in Subsection 3.4.

3.2 Air consumption rate

One breath by the Princess constitutes an oxygen intake corresponding to only 5% of the total volume of air she inhaled. This could be expressed as follows:

$$Q_{\text{oxy}} [\text{L/h}] = Q_{\text{air}} [\text{L/h}] \times 0.05. \quad (2)$$

Here Q_{oxy} is the amount of oxygen that the Princess intakes per hour, which she uses for energy production. Human cells use oxygen to create the energy source (i.e., ATP) through an oxygen-based chemical reaction to fuel cell functions. The amount of energy produced by the intake of 1 L oxygen is estimated to be 4.83 kcal/L [10]. We thus have

$$E_p [\text{kcal/h}] = 4.83 [\text{kcal/L}] \times Q_{\text{oxy}} [\text{L/h}], \quad (3)$$

where E_p is the rate of energy production following the intake of oxygen at the rate of Q_{oxy} . This is the energy consumed by the body for the maintenance of physiological life activities.

It is interesting to note that the consumption energy by human and many kinds of mammals is proportional to the total body surface area [11]. This is primarily because many mammals regulate their body temperature by heat exchange through the skin surface and the respiratory tract to maintain a constant body temperature at an extremely narrow range (e.g., 36–37°C for humans). Especially at rest, most of this heat exchange occurs through the skin surface. Therefore, it would be easy to determine there is a high degree of correlation between the body surface area and the energy consumption rate of the human body. The value of the proportionality constant is determined by the degree of active motion of the body, as listed in Table 1.

For instance, when a person is sitting still the energy consumption per hour per surface area is 50 kcal/(m² · h). Accordingly, the energy E_c consumed per hour can be calculated as follows:

$$E_c [\text{kcal/h}] = 50 [\text{kcal/(m}^2 \cdot \text{h)}] \times A [\text{m}^2], \quad (4)$$

where $A [\text{m}^2]$ is the total surface area of the body. In the

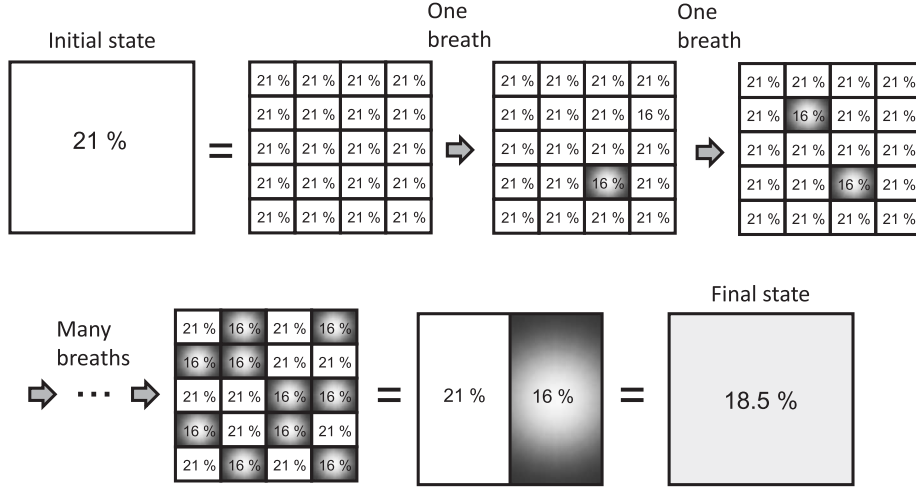


Fig. 3. Schematic diagram of the time variation in the oxygen concentration inside a bamboo chamber.

present study, we assumed that the energy was produced only by breathing and, consequently, $E_c = E_p$.

From Eqs. (2)–(4), we obtain

$$Q_{\text{air}} [\text{L/h}] = \frac{50 [\text{kcal}/(\text{m}^2 \cdot \text{h})] \times A [\text{m}^2]}{4.83 [\text{kcal/L}] \times 0.05}. \quad (5)$$

Therefore, the remaining task is the determination of the body surface area (A) of the Princess, which will be solved in the next subsection.

3.3 Total surface area of the human body

Developing methods for estimating the surface area of the human body has long been an important subject in the field of biology, physiology, medicine, and engineering [12]. It is currently accepted that the total surface area A [m^2] of the human body generally obeys a power law with respect to the height H [m] and weight M [kg] of the body as provided by [13]

$$A = 0.202 \times M^{0.425} \times H^{0.725}. \quad (6)$$

Let us use this power law to evaluate A of Princess. First, we remind that the body height of Princess Kaguya was assumed to be $H = 9.10$ cm [4]. Next, we considered the weight of the Princess, which was finally determined by assuming a similar relationship between H and M based on different scales as explained below. In Japan, the average height (H_0) and weight (M_0) of 5-years-old girls are $H_0 = 107.87$ cm and $M_0 = 17.6$ kg, respectively [14]. Assuming the similarity relationship, therefore, the body weight of the Princess, M , is

$$M = M_0 \times \left(\frac{H}{H_0} \right)^3 = 10.6 \times 10^{-3} [\text{kg}]. \quad (7)$$

It thus follows that

$$A = 5.14 \times 10^{-3} [\text{m}^2]. \quad (8)$$

In addition, the volume of Princess' body V_{Pr} can be deduced from Eq. (7) as

$$V_{\text{Pr}} = 10.6 [\text{cm}^3], \quad (9)$$

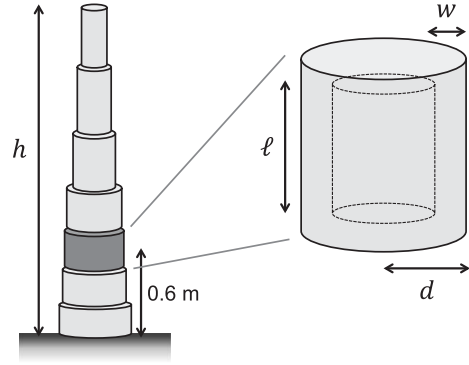


Fig. 4. A bamboo culm with the height of h and the internode located 0.6 m above the ground. The parameters ℓ , d , w used in the text are defined.

where the specific gravity of the Princess's body was assumed to be equal to that of water: 1 g/cm^3 .

3.4 Inner volume of a bamboo chamber

If the bamboo chamber is cylindrical with no tapering, the its inner volume, V_{inn} , could be calculated as follows:

$$V_{\text{inn}} = \frac{\pi}{4} \ell (d - 2w)^2, \quad (10)$$

where ℓ is the internode length, d is the outer diameter of the cross section, and w is the wall thickness of the bamboo culm. Figure 4 shows the schematics of the three geometric parameters and that of the total height (h) of a bamboo culm. We hypothesize that the Princess was in a chamber with a height of 0.6 m from the ground. This would be a reasonable height for an elderly man to cut with an axe or sickle while stooping. Furthermore, since the internode length decreases as it approaches the ground from above [15], those located lower than 0.6 m in altitude would tend to have had insufficient volume to enclose the Princess with a height of 9.10 cm.

To quantify V_{inn} of the internode chamber depicted in Fig. 4, we had to specify the species of bamboo in which the Princess was trapped. There were two possible candidates of bamboo species, which are listed in Table 2 and commonly called "Madake" and "Hachiku" in Japan [16].

Table 2. Measurement data of the bamboo culm sizes.

Species	<i>Phyllostachys bambusoides</i> Sieb. et Zucc. (Madake)	<i>Phyllostachys nigra</i> var. <i>henonis</i> (Hachiku)
h [m]	13.9	8.3
ℓ [cm]	20	20
d [cm]	6.2	3.8
w [cm]	0.74	0.65

These two species have existed in Japan for at least 800 years since they were domesticated in the late 9th century to the early 10th century when the tale of Princess Kaguya is thought to have been written. Most of the other bamboo species in Japan can be excluded as potential candidates because they were introduced in Japan only a few centuries ago [17] or they would not have had a sufficient volume to confine the Princess with a 9.10 cm height. Based on this background, we focused on the two bamboo species that have existed from ancient times (see Table 2), and actually personally collected several samples from the forest (similar to what the bamboo cutter did) to measure the ℓ , d , and w values using Eq. (10). The internode was measured at a height approximately 0.6 m from the ground.

Using the measurement values listed in Table 2 we used the following equation for the subsequent calculation:

$$V_{\text{inn}} = 3.5 \times 10^2 [\text{cm}^3] \text{ for Madake,} \quad (11)$$

$$V_{\text{inn}} = 98 [\text{cm}^3] \text{ for Hachiku.} \quad (12)$$

4. Results

Substituting the results obtained thus far into Eq. (1), we arrived at the conclusion that the viable time for Princess Kaguya could be calculated as follows:

$$T = 0.16 [\text{h}] = 9.6 [\text{min}] \text{ for Madake,} \quad (13)$$

$$T = 0.041 [\text{h}] = 2.5 [\text{min}] \text{ for Hachiku.} \quad (14)$$

In both cases above, the viable time was found to be less than 10 min, which appears to be surprisingly short. In fact, the situation allowed of no delay; the life of the Princess would have been in danger if the old man had not rescued her promptly. This observation would create the impression that she was an extremely fortunate or blessed girl.

5. Biophysical Scaling Relation

From the viewpoint of biophysics, the shortness in T given by Eqs. (13) and (14) is regarded as an important consequence of an allometric relation deduced from Eq. (1). Note that in the right-hand side of Eq. (1), the volume difference in numerator (i.e., $V_{\text{inn}} - V_{\text{Pr}}$) scales in cubic manner to the length dimension and the dominator (i.e., Q_{air}) scales in square manner, respectively. Therefore, the viable time T was proportional to the characteristic length of the system. Specifically, the smaller the body size and trapping space, the shorter the survival time of the live organism therein,

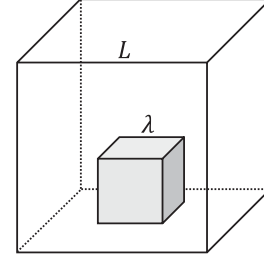


Fig. 5. Cubic-shaped closed space with a side length of L that contains a cubic-shaped living organism with λ as the linear dimension.

although the ratio of the both sizes remains unchanged (i.e., even when the geometric similarity holds).

The above discussion can be clarified by Fig. 5, which presents a simplified situation in which a cube with length of side λ is confined in a cubic cavity with length of side L . Using the inner cube to simulate a living organism, we could obtain the equations: $V_{\text{inn}} = L^3$, $V_{\text{Pr}} = \lambda^3$, $Q_{\text{air}} \propto \lambda^2$, which implied $T \propto (L^3 - \lambda^3)/\lambda^2$. If L/λ is fixed as a constant c , the following equation is obtained:

$$T \propto \frac{\lambda^3(c^3 - 1)}{\lambda^2} \propto \lambda. \quad (15)$$

Equation (15) reveals that the viable time is proportional to the characteristic body length λ when the geometric similarity $L/\lambda \equiv c$ is satisfied.

6. Summary

In this study, we estimated the time for which Princess Kaguya could have survived in the completely airtight bamboo chamber. Based on the assumed sizes of the Princess' body and bamboo chamber, we concluded that the duration was less than 10 min, which was shorter than what would be intuitively expected. The short duration is attributable to the allometric scaling, which implies that the viable time duration is linearly scaled to the characteristic length scale of the system.

As a final remark, we emphasize that the problem addressed is not limited to a fictitious “miniature-girl” situation, but can be extended easily to different situation in the body size, the amount of activity, and the number of persons trapped in a sealed space. Those extended problems will provide good exercises for the study of scaling relation between biological quantities.

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