Information Reduction for Chaotic Patterns

Yoshiki Hidaka^{1*}, Kosuke Ijigawa¹, Seung-Yong Kwak¹, Noriko Oikawa^{2†}, Hirotaka Okabe¹ and Kazuhiro Hara¹

¹Department of Applied Quantum Physics and Nuclear Engineering, Faculty of Engineering, Kyushu University, Fukuoka 819-0395, Japan ²Department of Physics, Graduate School of Science and Engineering, Tokyo Metropolitan University, Tokyo 192-0397, Japan *E-mail address: hidaka@ap.kyushu-u.ac.jp

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To investigate the universality and diversity of spatiotemporal chaos, information reduction, which describes phenomena using generalized quantities such as amplitude and phase, is an important technique. Several methods of image analysis are presented for information reduction of experimental image data of spatiotemporal chaos. **Key words:** Spatiotemporal Chaos, Image Analysis, Information Reduction

1. Introduction

Some of the dynamical properties of dissipative structures appearing in nonequilibrium open systems are universal among systems despite the constituent materials and the underlying mechanisms for structure formation being completely different. A method by which to explore the universality is information reduction.

One phenomenon that exhibits such universality is spatiotemporal chaos appearing in spatially-extended systems. The order of a dissipative structure is represented by a periodic function of a physical quantity. For example, in thermal convection systems, the physical quantity corresponds to the vertical component of the flow velocity. In the parameter range close to the occurrence point of the dissipative structure, long-wavelength instability is often generated by retaining local order. If this instability occurs as a macroscopic stationary fluctuation, it is called spatiotemporal chaos [1,2]. This fluctuation is represented by the slow spatiotemporal variations of the amplitude and phase of the periodic function. In this way, describing phenomena using generalized quantities such as amplitude and phase is called information reduction. With such generalizations, it is possible to compare nonlinear phenomena with different physical backgrounds and to analyze their diversity as well as universality.

In theoretical studies on dissipative structures, the equations governing amplitude and phase can be obtained from the fundamental equations, such as the Navier–Stokes equation for fluid systems and the reaction-diffusion equation for systems of chemical reactions, using the mathematical method called reductive perturbation. The Newell– Whitehead equation, the complex Ginzburg–Landau equation, and the Kuramoto–Sivashinsky equation were obtained in this manner [3].

In regard to experimental studies of spatiotemporal

chaos, the electroconvection that occurs in nematic liquid crystals on applying an electric field has played an important role, because it has the following experimental merits. Because the control parameter is the applied voltage, it is easy to precisely tune the control parameters necessary to generate spatiotemporal chaos. From the optical properties of the liquid crystals, no special technique is required to visualize the convection structure. Using an optical microscope, one can observe the spatiotemporal structure of convection as a temporal change u(x, y, t) in the gray-scale value of a two-dimensional (2D) image^{*1}. One can observe large systems that exclude cumbersome influences from lateral boundaries with time scales suitable for the experiments.

Image analysis is an important tool to enable a further reduction of the spatiotemporal chaos observed as a timedependent pattern u(x, y, t). In this article, we introduce several methods of image analysis that provide information reduction for three types of spatiotemporal chaos appearing in nematic electroconvection [2].

2. Soft-Mode Turbulence

Soft-mode turbulence is a type of spatiotemporal chaos in which the direction of the convection roll structure changes irregularly in time and space. A perfect static periodic pattern (called normal rolls) (Fig. 1) in which the convection rolls are parallel to the y-direction, that is, the wavevector is $\mathbf{q} = (q_0, 0)$, is expressed as

$$u(x, y, t) = R_0 \cos(q_0 x + \alpha_0).$$
(1)

In contrast, the essential part of soft-mode turbulence (Fig. 2(a)) with $\mathbf{q} = (q_x, q_y)$ can be expressed as

$$u(x, y, t) = R_0 \cos[q_x(x, y, t)x + q_y(x, y, t)y], \quad (2)$$

$$q_x(x, y, t) = q_0 \cos \psi(x, y, t),$$
 (3)

$$q_{y}(x, y, t) = q_{0} \sin \psi(x, y, t).$$
 (4)

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[†]Present address: Department of Physics and Electronics, Graduate School of Engineering, Osaka Prefecture University, Sakai 599-8531, Japan.

^{*&}lt;sup>1</sup>The basic equations of fluid dynamics for nematic liquid crystals are composed of functions of (x, y, z, t) of two vector quantities, flow velocity and molecular orientation. Therefore, at this point, the first-step reduction to u(x, y, t) is done.



Fig. 1. Normal rolls. White lines correspond to convective rolls.

Therefore, soft-mode turbulence can be described by $\psi(x, y, t)$ through information reduction.

The procedure to obtain $\psi(x, y)$ from a snapshot of softmode turbulence u(x, y) is described as follows. First, the bandpass filter using Fourier spectral analysis is applied to u(x, y) so that it approaches the form of Eq. (4). Specifically, after applying the Fourier transformation to u(x, y), the inverse transformation is applied to only those components with wavenumbers of magnitude of around q_0 . Figure 2(b) shows the image after applying the filter to (a). Furthermore, focusing on the fact that the convective periodic structure is kept locally, one can obtain $\psi(x, y)$ as follows. First, a small 2D square area s_2 centered at a point (x, y) is selected from the whole system. The size of ℓ on one side of s_2 is much smaller than the system size L and is sufficiently larger than the basic period, i.e., the size of a roll pair $\lambda_0 = 2\pi/q_0$. If a spot centered at $(q_0 \cos \psi_s, q_0 \sin \psi_s)$ appears in the 2D spectrum of u in s_2 , $\psi(x, y) = \psi_s$. One can obtain $\psi(x, y)$ in the whole system by finding ψ_s in this way by shifting s₂ ^{*2}. Figure 2(c) presents $\psi(x, y)$ obtained by this method. Since \mathbf{q} and $-\mathbf{q}$ cannot be distinguished, $\psi(x, y)$ is defined in $-90^{\circ} < \psi(x, y) \le 90^{\circ}$.

The problem of this method is that the resolution of ψ_s is low, because with the extraction of s_2 the wavenumber space is small. The method using the gradient of u(x, y) was adopted for other similar patterns [4]. The resolution of the result by this method is certainly high. However, desirable results for soft-mode turbulence cannot be obtained with this method, because it is difficult to find many necessary preprocessing and postprocessing tools.

3. Spatiotemporal Intermittency

In spatiotemporal intermittency, ordered regions coexist with turbulent regions. They are separated by clear boundaries that change with time. In nematic electroconvection, two types of spatiotemporal intermittency appear. One is a type that appears by partly collapsing the "defect lattice" into a turbulent state (Fig. 3(a)) [5,6], and the other is a type that appears by the partial change of turbulence into a "grid pattern" (Fig. 4(a)) [7]. The patterns of spatiotemporal in-



Fig. 2. (a) Soft-mode turbulence. (b) Image after applying the bandpass filter to (a). (c) Gray-scale plot of $\psi(x, y)$ obtained from (b). White and black correspond to $\psi = \pi/2$ and $-\pi/2$, respectively.

termittency can be described by the variable $\delta(x, y)$, which takes 1 in the ordered region and 0 in the turbulent region.

 $\delta(x, y)$ for the defect-lattice type is obtained by the following method [5,6]. A one-dimensional small area s₁ centered at a point (x, y) is selected from the whole system as a role pair corresponds to the basic period of the defect lattice. The pattern u(x') in s₁ is expanded as a Fourier series

$$u(x') = \sum_{q} \tilde{A}_{q} \exp(\mathrm{i}qx') + \mathrm{c.c..}$$
(5)

The amplitude $|\tilde{A}_{q_0}|$ of the wavenumber q_0 corresponding to a roll pair becomes large in the ordered region, and becomes small in the turbulent region. Therefore, if $|\tilde{A}_{q_0}| \ge a$, the small area s₁ is taken to be an ordered state. Here, *a* is an appropriate threshold. By performing such evaluations,

$$\delta(x, y) = \begin{cases} 1 & (|A_{q_0}(x, y)| \ge a) \\ 0 & (|\tilde{A}_{q_0}(x, y)| < a) \end{cases}$$
(6)

^{*&}lt;sup>2</sup>Actually, the spot is spread around $(q_0 \cos \psi_s, q_0 \sin \psi_s)$. The spread in the azimuthal direction is regarded as an error for ψ_s .



Fig. 3. Spatiotemporal intermittency of a defect-lattice type. (a) u(x, y). (b) $\delta(x, y)$ obtained from (a). The ordered area (defect lattice) where $\delta(x, y) = 1$ is colored white, and the turbulent area where $\delta(x, y) = 0$ is colored black.

with shift s_1 , $\delta(x, y)$ of the whole system can be obtained. Figure 3 shows a typical example of $\delta(x, y)$ of the defectlattice type. In contrast, for the grid-pattern type, a similar evaluation is made for a 2D small area, because the fundamental order structure is a section of the grid. Figure 4 shows a typical example of $\delta(x, y)$ of the grid-pattern type.

4. Defect Turbulence

Defect turbulence (Fig. 5(a)) is a type of spatiotemporal chaos in which topological defects are generated as a result of fluctuations of the normal rolls. This is regarded as the state in which the position of convection rolls in the normal roll is fluctuating in the *x*-direction. The pattern of defect turbulence can be described as

$$u(x, y, t) = R_0 \cos[q_0 x + \alpha(x, y, t)],$$
(7)

because the position of the roll is represented by the phase. Actually, the amplitude is also a function of (x, y, t). However, it can be regarded as the constant R_0 in Eq. (7), because the amplitude is nearly constant except that it becomes zero at a defect. Thus, important properties of defect turbulence are included in $\alpha(x, y)$. A topological defect generated as a consequence of roll fluctuations corresponds to a singular point of the phase $\alpha(x, y)$.

 $\alpha(x, y)$ can be obtained by a method called "complex demodulation". The details are described in Appendix



Fig. 4. Spatiotemporal intermittency of a grid-pattern type. (a) u(x, y). (b) $\delta(x, y)$ obtained from (a). The ordered area (grid pattern) where $\delta(x, y) = 1$ is colored white, and the turbulent area where $\delta(x, y) = 0$ is colored black.



Fig. 5. (a) Defect turbulence. (b) sin(x, y) obtained from (a). This is displayed in gray-scale with sin(x, y) = 1 as white and sin(x, y) = 0 as black.

[8]. By this method, R(x) and $\alpha(x)$ can be obtained from $u(x) = R(x) \cos(q_0 x + \alpha(x))$. One can obtain $\alpha(x, y)$ by calculating $\alpha(x)$ for all *y*, as evident from Fig. 5(b).

5. Discussion

Using the dynamics of these reduced variables $\psi(x, y, t), \delta(x, y, t),$ and $\alpha(x, y, t),$ comparisons of the present spatiotemporal chaos with well-known models can be performed [2]. $\psi(x, y)$ of soft-mode turbulence can be compared with the two-dimensional XY model (also called planar rotor model) [9]. The so-called Kosterlitz-Thouless transition occurs in the two-dimensional XY model [10], but order-disorder transition occurs in softmode turbulence [11]. $\alpha(x, y)$ of defect turbulence also can be compared with the two-dimensional XY model. In defect turbulence, a defect which corresponds to a singular point of $\alpha(x, y)$ is always generated due to the nonlinear phase instability [12]. Therefore, a no-defect state corresponding to the lower temperature phase below the Kosterlitz-Thouless transition point in the two-dimensional *XY* model cannot appear in defect turbulence. $\delta(x, y, t)$ of spatiotemporal intermittency can be compared with the directed percolation model [13]. The switching dynamics between the two states of $\delta(x, y, t)$ in spatiotemporal intermittency of nematic electroconvection is not a Poisson process [7]. This behavior is different from that in the directed percolation. Thus, differences generated from the model reveal the properties inherent in each type of spatiotemporal chaos.

Furthermore, we can try to create phenomenological models for the spatiotemporal chaos using these variables. These models may present new perspectives for spatiotemporal chaos.

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Appendix A.

From a one-dimensional profile

$$u(x) = R(x)\cos[q_0 x + \alpha(x)], \qquad (A.1)$$

R(x) and $\alpha(x)$ are demodulated in the following way.

Equation (A.1) is rewritten as

$$u(x) = \operatorname{Re}\{R(x) \exp[i\alpha(x)] \exp(iq_0 x)\}$$

= Re[A(x) exp(iq_0 x)]
= $\frac{1}{2}[A(x) \exp(iq_0 x) + A^*(x) \exp(-iq_0 x)](A.2)$

with $R(x) \exp[i\alpha(x)] \equiv A(x)$. X* is the complex conjugate of X. The Fourier transform of u(x) of Eq. (A.2) is

$$J_{0}(k) = \int_{-\infty}^{\infty} u(x) \exp(-ikx) dx$$

= $\frac{1}{2} \left\{ \int_{-\infty}^{\infty} A(x) \exp[-i(k-q_{0})x] dx + \int_{-\infty}^{\infty} A^{*}(x) \exp[-i(k+q_{0})x] dx \right\}$
= $\frac{1}{2} [B(k-q_{0}) + B^{*}(-(k+q_{0}))],$ (A.3)

where B(k) is the Fourier transform of A(x). Low-pass filtering is applied to $J_0(k)$, because only the low wavenumber components are essential. After filtering, only the second term of Eq. (A.3) remains,

$$J(k) = \frac{1}{2}B(k - q_0).$$
 (A.4)

Using Eq. (A.4), the inverse Fourier transform of B(k), i.e., A(x), can be obtained as follows,

$$A(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(k) \exp(ikx) dk$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} 2J(k+q_0) \exp(ikx) dk$$

Therefore, with $J(k + q_0) \equiv J'(k + q_0) + iJ''(k + q_0)$,

$$A(x) = \frac{1}{2\pi} \left[2 \int_{-\infty}^{\infty} J'(k+q_0) \exp(ikx) dk + 2i \int_{-\infty}^{\infty} J''(k+q_0) \exp(ikx) dk \right]$$

= 2[I'_r(x) + iI'_i(x) + iI''_r(x) - I''_i(x)], (A.5)

where $I'_{\rm r}(x)$ and $I'_{\rm i}(x)$ are the real and imaginary parts, respectively, of the inverse Fourier transform of $J'(k+q_0)$. Similarly $I''_{\rm r}(x)$ and $I''_{\rm i}(x)$ are the real and imaginary parts of the inverse Fourier transform of $J''(k+q_0)$. Finally, from Eq. (A.5),

$$R(x) = \sqrt{[\text{Re}A(x)]^2 + [\text{Im}A(x)]^2}$$

= $2\sqrt{[I'_r(x) - I''_i(x)]^2 + [I'_i(x) + I''_r(x)]^2}$

and

$$\alpha(x) = \arctan\left[\frac{\mathrm{Im}A(x)}{\mathrm{Re}A(x)}\right]$$
$$= \arctan\left[\frac{I_{i}'(x) + I_{r}''(x)}{I_{r}'(x) - I_{i}''(x)}\right]$$

can be obtained.

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