The Longitude Problem as the Unification of Space and Time with Special Application to the Island of St Helena

Takeshi Sugimoto

Department Information Systems Creation, Kanagawa University, Yokohama, Japan
E-mail address: sugimt01@kanagawa-u.ac.jp

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Determination of the longitude is the unification of space and time. The course note is presented to understand the points in the longitude problem within the framework of 16–8 Centuries. Establishing local time is difficult, because there are leads and lags of the Sun's culminations, known as 'Equation of Time.' The Sun's motion in the celestial sphere is nonlinear because of geometrical and gravitational reasons. The novel algorithm is proposed to get the rigorous Equation of Time. Three brilliant astronomers had stayed in St Helena. They themselves determined the longitude of the island by different methods. By use of their and other observations of astronomical events the new calculations are available. These are the sources of the exercises. Edmond Halley states the island in 7 degrees west of London. There are three observations of the Transit of Mercury on 7th November 1677: 5°53′21″W by Towneley data and 6°26′44″W by Avignon data. Nevil Maskelyne determined the longitude to be 5°49′W based on eclipses of Jupiter's satellites in 1761. Using observations of the total lunar eclipse on 18th May 1761 at the island, Cape and Stockholm, one obtains 5°49′05″W. Manuel John Johnson reached the conclusion of 5°43′39″W by the lunar distance method (1830–33). Using observations of the total solar eclipse on 27th July 1832 at the island, one obtains 5°27′51″W.

Key words: Lunar Distance Method, Jupiter's Satellites, Transit of Mercury, Eclipses

1. Introduction

Determination of the longitude is the unification of space and time. We prepare the course (text) of understanding the longitude problem within the framework of 16-8 Centuries. We also prepare exercises. Three brilliant astronomers had stayed in St Helena. They themselves determined the longitude of the island by different methods. By use of their and other observations of astronomical events the new calculations are possible. These are the sources of the exercises.

1.1 Common reckoning

At the dawn of 'Age of Exploration' Iberian sailors voyaged far and far away from their mother lands with charting instruments. Some voyages are successful with many discoveries and ethnic souvenirs. But there are many shipwrecks. Time on a ship is measured by marine sandglass, which is hourglass. To know the latitude a pilot uses an astrolabe to measure altitudes of the Sun or stars. But at sea there was no way to know the longitude. Instead pilots uses 'common reckoning' that is a kind of bearing. Figure 1 shows the basic kit for common reckoning. Drop the log-chip into the wake of the ship, and a pilot and his assistants measure the length (knots) of the rope tied to the log-chip in given seconds (sandglass); one more thing to do is to know the direction of the tight rope. After simple algebra the pilot knows the velocity vector of the ship. On the marine chart the pilot draws a line, which shows the course of their ship. This bearing at sea has a setback. Without knowing ocean current vectors, this pilot misleads the ship. Below we see such a faulty result.

A wooden log-chip is tied to a rope. The rope is graduated by knots and winded up around a reel. One knot is equal to 7 or 8 fathoms depending on pilot's choice. A small sandglass is for measurement of given seconds, say less than 30 seconds.

1.2 Jupiter's satellites

Longitude corresponds to local time difference: 15 degrees = one hour. Suppose there are two astronomers in remote places with different local time. If these astronomers observed the distinctive celestial event at the same time, comparison of local time leads to the time difference, i.e., the longitude difference. Below we show examples of use of the transit of Mercury, the lunar eclipse, and the solar eclipse. But these events are not so frequent and not that convenient to determine the longitude. By the way Galileo Galilei is the first to use a telescope for celestial observations, and Galileo finds four Jupiter's satellites in 1610.

His presentation is a precursor of modern 'satellite tracks' in astronomical almanacs (Fig. 2).

Discovery of Jupiter's satellites has two meanings: to let the world know the existence of the third center of revolution; this fact is a big blow to the geocentric theory, because theorists ridiculed the heliocentric theory with 'two centers' of revolution. Galileo knew eclipses of satellites could be used as celestial events, which in turn serves to solve the longitude problem. This is another meaning of the discovery. Periods of satellites vary from a couple of days to half a month. By the method using Jupiter's satellites Galileo

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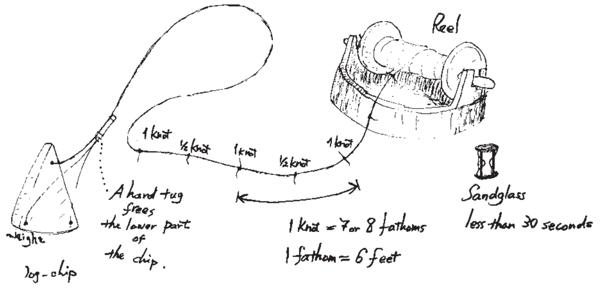

Fig. 1. Common reckoning kit.

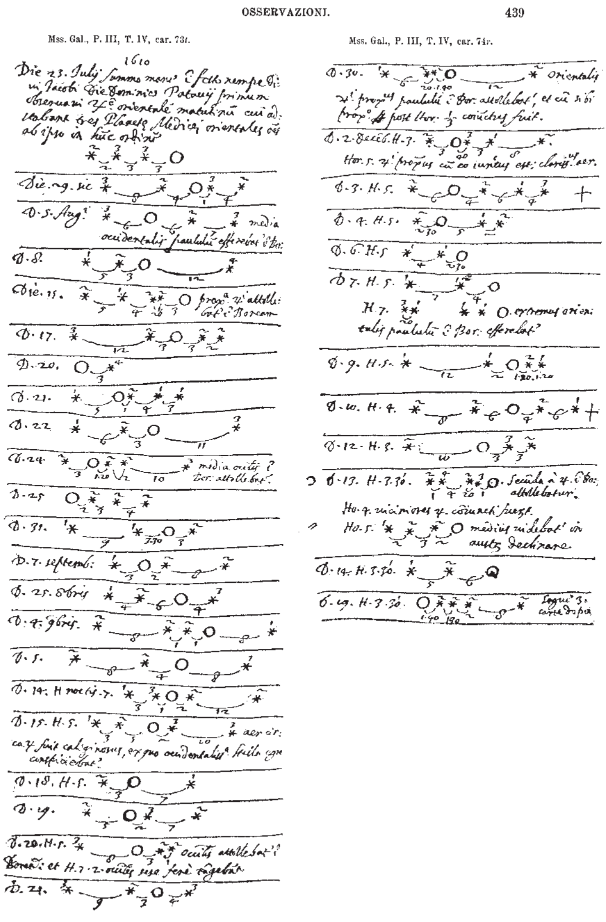

Fig. 2. Galileo's manuscript about Jupiter's satellites [1].

applied to a couple of prizes for the longitude problem, but his efforts were not successful. One reason is difficult to observe Jupiter's satellites at sea, and another reason is inaccuracy in his predictions of eclipses. Galileo does not believe in Kepler's laws. The method of Jupiter's satellites had been refined by many astronomers in 17 Century. The use 'on land' became practical in the end.

1.3 Outline of the work

Section 2 is the main source of education. Subsection 1.1 is for consideration of local time. Establishing local time is not an easy task at all. A sundial is not enough. We will describe what to do. Within the framework of Kepler problem, we need to obtain the nonlinear behavior of the Sun's culminations, i.e., 'Equation of Time.' After setting up local time, we are prepared to determine the local longitude by astronomical observations. Three astronomers' stories offer good examples of exercising determination of longitude. Subsection 1.2 is for Edmond Halley (1656–1742) in the island of St Helena (1677–8). Our exercise makes most of his observation of the transit of Mercury on 7th November 1677 (New Style) to determine the longitude of the island by comparing with the continental observations. Subsection 1.3 is for Nevil Maskelyne (1732–1811) in the island (1761–2). He compared the common reckoning and the lunar distance method on the ongoing voyage to estimate longitude at sea. He used the method of Jupiter's satellites to determine the longitude of the island. Our exercise makes use of the lunar eclipse on 18th May 1761. Subsection 1.4 is for Manuel John Jonson (1805–59) in the island (1823–33). He used the lunar distance method to determine the longitude of the Ladder Hill observatory in the island. We mention how to use 'Nautical Almanac.' Our exercise makes use of the solar eclipse on 27th July 1832. Section 3 is for our findings.

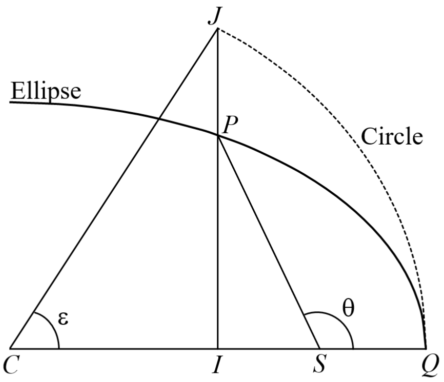

Fig. 3. Definitions of the three anomalies.

2. Philosophy and Practices

2.1 Establishment of local time

The frame work is called Kepler problem that is the world ruled by the law of gravitation with neglecting mass of a planet compared to that of the Sun. Apparently motion of the planet is faster near the perihelion and slower near the aphelion than the mean motion. Another source of difficulty lies in the fact that the rotational axis of the earth is tilted to the revolutionary axis around the Sun. Because of this geometrical reason the Sun culminates at noon only four times a year. We call the deviation from the mean motion of the Sun, i.e., the calendar and the clockwork, 'Equation of Time.'

The orbit of a planet is an ellipse; C is its center; the Sun sits at S (focus); Q is the perihelion; the planet is now at P; CQJ is a sector of a circle with its radius equal to the semi-major axis of the ellipse; the straight segment IPJ is perpendicular to the center line QSC; the angle ϵ is the eccentric anomaly; the angle θ is the true anomaly; the non-dimensional sectorial-area SQP is the mean anomaly (M).

To derive Equation of Time (EOT) we introduce three anomalies: the true, the eccentric, and the mean anomalies. Please refer to Fig. 3 for definitions of these anomalies.

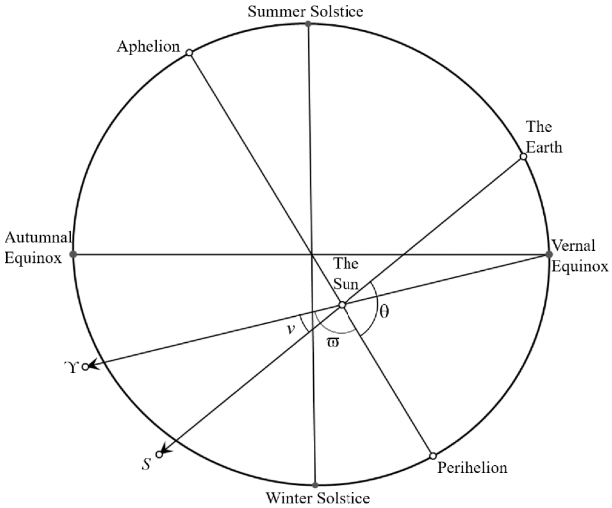

Fig. 4. The orbit of the Earth.

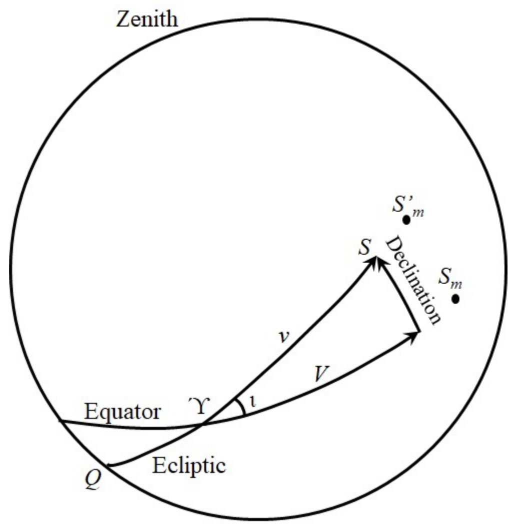

Fig. 5. The relation between space and time.

The true anomaly θ is the angle between the perihelion-Sun orientation and the planet-Sun orientation. Kepler recognizes an ellipse as a circle squeezed vertically by the ratio of (minor axis)/(major axis). Therefore he introduces a co-centric circle with a radius = the semi-major axis of the ellipse. The point J is the projection of the planet position P upon the co-centric circle. The eccentric anomaly ϵ is the angle between the perihelion-center orientation and the projection-center orientation. The mean anomaly M is the non-dimensional sectorial-area spanned by the perihelion Q, the Sun S, and the planet P. Because of Kepler's second law M is in proportion to time. Kepler finds that M is given by subtracting the area of ΔIJC from the area of the circular sector QJC, multiplying by (minor axis)/(major axis), and then subtracting the area of ΔSIP, where C and I denote the center of the ellipse and the circle and the foot of the perpendicular JP upon the center line, respectively. The result is given by

$$M = \epsilon - e \sin \epsilon,$$

where e denotes the eccentricity of the ellipse. This is called Kepler's equation, which represents the gravitational effect.

The anomalies ϵ and θ are related by

$$\tan \theta/2 = \sqrt{(1+e)/(1-e)} \tan \epsilon/2.$$

Figure 4 shows a generic image with the exaggerated eccentricity. There are four seasonal nodes: the Vernal Equinox (Υ), the Summer Solstice, the Autumnal Equinox, and the Winter Solstice. There are two extreme positions: the perihelion closest to the Sun and the aphelion farthest from the Sun. Notations are as follows: v is the ecliptic longitude; ϖ is the ecliptic longitude of the perihelion; θ is the true anomaly.

The conclusion from Fig. 4 is the geometrical relation below:

$$\varpi + \theta - v = \pi.$$

Apparently the Sum moves along the ecliptic on the celestial sphere. But our calendars and clockworks are graduated along the celestial equator. Figure 5 shows the relation between the ecliptic and the celestial equator. The angle between them is the obliquity ι. By use of spherical trigonometry we obtain the relation between the ecliptic longitude v, i.e., ΥS, and the corresponding right ascension V as follows:

$$\tan V = \tan v \cos \iota.$$

The mean motions are the same on the ecliptic and the celestial equator. We have the relation below:

$$\Upsilon S_m = \Upsilon S'_m.$$

Figure 5 is a celestial sphere; Q is the perihelion; Υ is the Vernal Equinox; S is the actual position of the Sun; v is the ecliptic longitude; V is the right ascension; the angle ι is obliquity of the ecliptic; S'_m is the mean position of the steadily-moving Sun on the ecliptic; S_m is the mean position of the steadily-moving Sun on the celestial equator.

Obtaining all the necessary relations, we define the equation of time by

$$\Upsilon S_m - V.$$

We derive a more convenient form after some algebra:

$$\begin{aligned}\Upsilon S_m - V &= \Upsilon S'_m - \Upsilon S + v - V \\ &= QS'_m - QS + v - V \\ &= M - \theta + v - V \\ &= M - \pi + \varpi - V.\end{aligned}$$

[The algorithm for EOT]

(1) Give the ecliptic longitude v.
Note $v = 0$ at Υ.
(2) Get the right ascension by
$V = \text{atan}(\tan v \cos \iota)$.
(3) Get the true anomaly by
$\theta = v + \pi - \varpi$.
(4) Get the eccentric anomaly by
$\epsilon = 2\text{atan}(\sqrt{(1-e)/(1+e)} \tan \theta/2)$.
(5) Get the mean anomaly by
$M = \epsilon - e \sin \epsilon$.

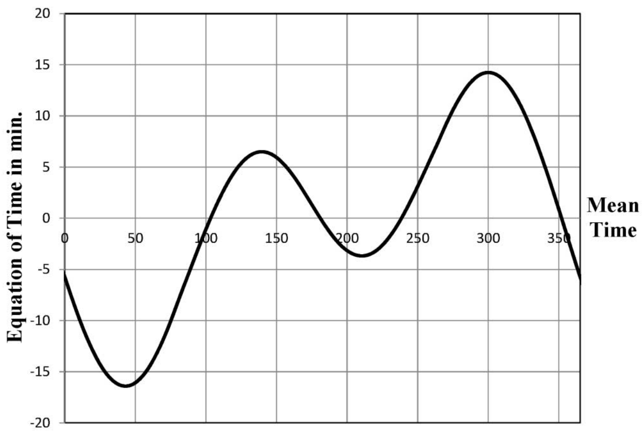

Fig. 6. Equation of Time vs Mean time.

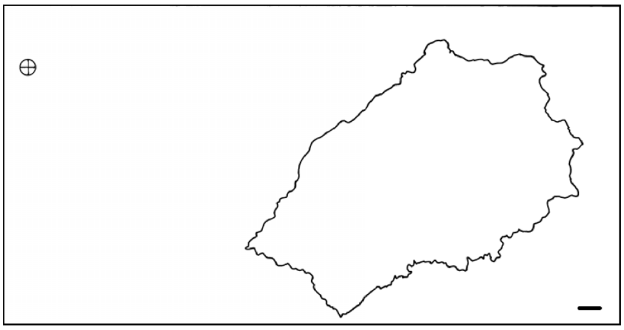

Fig. 7. The estimated position by use of Towneley data and the real position of the island; the bold bar corresponds to 1 km for Figs. 7–9.

(6) Evaluate the equation of time by
$M - \pi + \varpi - V$.
(7) Repeat (1)–(6) for all year around.

Figure 6 shows EOT in min plotted against 365 days from January to December (see Appendix for parameters used).

Now you can establish the local time with your observation by a sundial and correction due to the equation of time.

2.2 Edmond Halley's observations in the island of St Helena

He left Downs on 3rd December 1676 and arrived at the island in March 1677. He charted the southern stars. He left the island in March 1678 and arrived at England in May. He published the results of his expedition as 'Catalogus Stellarum Australium' in 1679 [2]. On its title page Halley states 'the island of St Helena (Latitude: 15°55′ S; and Longitude: 7 degrees west from London).' The rare celestial events are useful to determine the longitude. That is the transit of Mercury on 7th November 1677. Halley observed the ingress and the egress.

There are two European observations about the same transit. In Towneley, England, Richard Towneley (1629–1707) observed the egress [3]. In Avignon, France, Jean Charles Gallet observed almost all the process [4]. In all the observations we learn only data on the exterior contact of the egress, the emersion, are common. We make use of the today's longitudes of Towneley Hall and St Symphorien Church. The raw data are as follows:

//time of emersion//	//longitude//
*Avignon $15^h26^m56^s$	4°48′46″ E
*Towneley $14^h56^m36^s$	2°13′21″ W
*St Helena $14^h41^m54^s$	'to be determined'

Translating the time difference into the longitude difference, we determine the longitude of St Helena:
6°26′44″ W by use of Avignon data;
5°53′21″ W by use of Towneley data.

The latter is better, but this estimate points the place one island away from St Helena. The clock of Avignon gains two minutes or so.

2.3 Nevil Maskelyne's observations in the island of St Helena

He left Portsmouth on 17th January 1761, and arrived at the island on 6th April. He failed to observe the transit of Venus on 6th June. The malfunction of his zenith sector let him abandon the annual observation of Sirius. He left the island on 19th February 1762, and arrived at Downs on 7th June.

He had systematically examined the lunar distance method on both ongoing and return voyages. On the way to the island he also tried the common reckoning. He published all these efforts as 'British Mariner's Guide' in 1763 [5]. In this book Maskelyne states 'the longitude, by the common reckoning, was 1°28′ east of London.' Thus the common reckoning misleads us to the disastrous result, as we mention above. The cloudiness of the island also annoyed Maskelyne. He could observe the Moon's culmination only once, so he abandoned to use the lunar distance method to determine the longitude of the island. Instead he observed eclipses of Jupiter's satellites, and he arrived at the conclusion: 5°49′ west of Greenwich [5]. Maskelyne determined the latitude to be 15°55′S.

Our exercise makes use of the lunar eclipse on 18th May 1761. Nevil Maskelyne [6], Mason and Dixson at the Cape of Good Hope [7], and Peter Wargentin at Stockholm [8] recorded the emersion and the end of eclipse in common:

//location//	//emersion//	//end of eclipse//	//longitude//
*St Helena	$10^h39^m23^s$	$11^h46^m52^s$	to be determined
*Cape	$12^h15^m37^s$	$13^h23^m42^s$	$1^h13^m35^s$ E
*Stockholm	$12^h15^m00^s$	$13^h21^m08^s$	$1^h12^m01^s$ E

Taking the mean of two chances, we determine the longitude of St Helena:
5°44′15″ W by use of Cape data;
5°53′55″ W by use of Stockholm data.

The estimate by Cape is pretty good, but the mean of these gives us
5°49′05″.

This is much the same as Maskelyne's estimation. The estimated position is the place half an island away from the real position.

2.4 Manuel J. Jonson's observations in the island of St Helena

In 1823 Johnson came to the island as Lieutenant of the East-India Company Artillery. The Governor and General Alexander Walker ordered Johnson to establish an astronomical observatory, and he founded it on Ladder Hill in

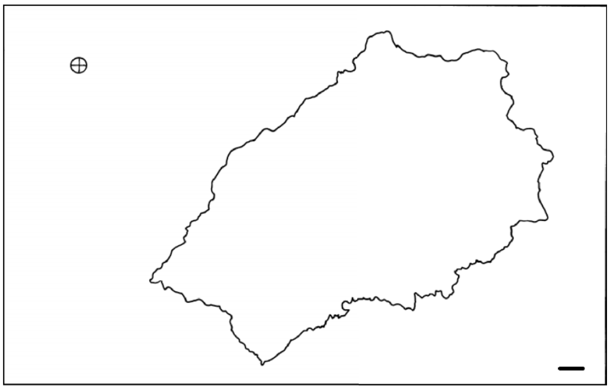

Fig. 8. The position estimated by Maskelyne and the real position.

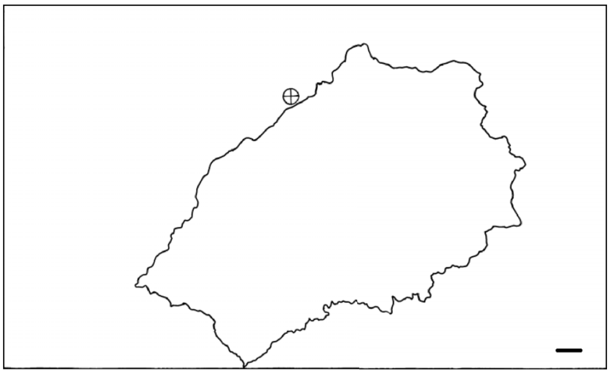

Fig. 9. The position estimated by Johnson and the real position.

1826. Johnson had made observations of the southern stars from November 1829 to April 1833. In 1833 he retired from the Artillery and went back to England. There in 1835 he published his work as the star catalogue [9], by which he received the Gold Medal from the Royal Astronomical Society. The year 1835 saw the return of Halley's Comet.

The lunar distance method is shortly to know the Moon's position in the celestial sphere. An astronomer or a navigator observes the culmination of the Moon and compare the data with the master data of the Sun or the fixed star predicted in 'Nautical Almanac [10].' After some algebra one gets the longitude of the observed place from Greenwich. There is the companion manual [11] that comprehensively describes how to handle data. The center of the Moon is merely conceptual, because the Moon waxes and wanes. The observation is made at the Moon's limbs. One calculates when the conceptual center of the Moon culminates. Johnson determined the longitude of the Ladder Hill observatory by the lunar distance method comparing his 1830–33 data with those at Greenwich, Cambridge and the Cape of Good Hope. The result is as follows:

The latitude: $15°55'26''$S; the longitude: $5°43'39''$W.

As shown in Fig. 9, the estimated position is in the ocean and several hundred metres away from the observatory site.

There was a solar eclipse on 27th July 1832. Jonson observed this solar eclipse at the Ladder Hill observatory. But the solar eclipse is not a global event, and he estimates timings of the conjunction [11] by using data observed at St Helena ($13^h39^m8.6^s$), St Fernando near Cadiz ($13^h37^m2.5^s$), Marseille ($14^h23^m19.4^s$), and Padua ($14^h49^m11.4^s$). Comparing differences of the conjunction timings, he arrives at the conclusion: by St Fernando near Cadiz $5°40'45''$W; by Marseille $5°40'27''$W; by Padua $5°38'24''$W. Johnson is not satisfied with these longitude estimates.

Our exercise makes use of the true conjunction time at Greenwich. It is, however, unavailable, so we shall use NASA data base: $14^h01^m00^s$ (UT) with 6 seconds error.

*St Helena: $14^h01^m00^s - 13^h39^m8.6^s = 21^m51.4^s$.

This is very short, for the estimated longitude is only $5°27'51''$W. Let us check out the other sites:

*St Fernando: $14^h01^m00^s - 13^h37^m2.5^s = 23^m57.5^s$; longitude at $24^m49.1^s$ by [10];

*Marseille: $14^h01^m00^s - 14^h23^m19.4^s = -22^m19.4^s$; longitude at $-21^m29.0^s$ by [10];

*Padua: $14^h01^m00^s - 14^h49^m11.4^s = -47^m11.4^s$; longitude at $-47^m29.2^s$ by [10].

The longitudes in [10] are quite correct, so the estimated timings of the conjunction is the source of the errors. The errors of these sites are 51.6^s (St Fernando), 50.4^s (Marseille), and 42.2^s (Padua). It is difficult to determine the longitude by the solar eclipse, because the observation depends on the location of the earth.

3. Conclusion

In the Age of Exploration the pressing problem is determination of the longitude at sea or on an unknown place. Before the marine chronometers became commodities among navigators, the astronomical observations are crucial clues to determination of the longitude. The process is the unification of space and time: to read the celestial situation and the local time; then to compare the local readings with those on the mother land.

In the first place we point out establishing local time with the connection to 'Equation of Time,' the phenomena geometrical as well as gravitational. The solar culmination must be corrected by 'Equation of Time.' The algorithm for 'Equation of Time' presented here is our novel and rigorous recipe.

The next step is observing the celestial events that would be timing-sensitive. Historically astronomers paid attention to eclipses of Jupiter's satellites. By mid-17th Century this became the most accurate method, but setbacks are inapplicability at sea, necessity of skills in observation, and sometimes unavailability.

The second best is the lunar distance method. This requires comparison between the lunar culmination and the solar culmination (or other star culmination). Less skilled navigators could observe the large Moon. But the Moon waxes and wanes. Therefore one must observe the Moon by its limbs, and this fact affects accuracy.

There are introduction of three other ways of determining the longitude and accompanying exercises about the island of St Helena.

(1) Edmond Halley's observation of 'the transit of Mercury:' the result points the place one island away from St Helena.

(2) Nevil Maskelyne's observation of the total lunar eclipse: the result points the place half an island away from St Helena.

(3) Manuel John Johnson's observation of the total solar eclipse: the result is the worst (much too short); the linear method of estimating the conjunction is the source of errors.

Appendix A.

Parameters used in the algorithm are given as follows: $\varpi = 1.3456$ [rad]; Υ on 23 March in a non-leap year; $e = 0.0167$; $\iota = 0.4089$ [rad].

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