

Full-model Finite-element Analysis for Structural Color of *Tarsiger cyanurus*'s Feather Barbs

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This paper presents full modeling of the cross-section of a blue feather barb of *Tarsiger cyanurus* and computed optical properties of the barb by means of finite element simulations. Cross-sectional models of the blue barbs are created based on SEM and TEM observations and are characterized on the basis of the radial sizes and the arrangements of air-pores. Large-scale finite element method is employed in order to investigate reflective properties of a blue feather barb. Measured and computed results show good agreement in the visible wavelength range, when we take into account optical absorption of keratin.

Key words: *Tarsiger cyanurus*, Finite Element Method, Structural Color, Reflection Spectrum, Absorption

1. Introduction

Structural colors (Wiersma, 2013; Kinoshita and Yoshioka, 2005a, b; Vigneron *et al.*, 2006) are observed in many living organisms as *Morpho* butterflies (Kinoshita *et al.*, 2002a; Yoshioka and Kinoshita, 2004; Yoshioka and Kinoshita, 2005; Ingram and Parker, 2007) the butterfly *Curetis acuta* Moore (Liu *et al.*, 2019), peacock feathers (Kinoshita and Yoshioka, 2002; Zi *et al.*, 2003), earthworm (Ueta *et al.*, 2014b), *Polia* fruit (Vignolini *et al.*, 2012) and so on (Shevtsova *et al.*, 2011; Saranathan *et al.*, 2012), and have been investigated vigorously in order to clarify the origin of the twinkling fascinating colors. The colorations are caused by optical scattering and interference in nano-scale structures, so called biological photonic crystals (Vukusic and Sambles, 2003; Vukusic and Stavenga, 2009). The structural colors are different from the pigmentary colors of materials composing the structures, and the coloration is generally determined by the structure and the spatial distribution of the dielectric constant of materials.

Numerical analysis is effective to understand how optical scattering and interference occur in the biological photonic crystals, and practically various analyses have been performed and have revealed the reasonable explanation of each case of structural color. Then, many kinds of numerical methods were employed. Especially, the finite difference time domain method (FDTD) is frequently used to analyze the structural color. *Morpho* butterfly is one of the most familiar target for FDTD analysis (Lee and Smith, 2009; Zhu *et al.*, 2009; Steindorfer *et al.*, 2012; Wang *et al.*, 2013) and the influence of structural disorder and the dependence on the incident angles of light have been investigated in the reflection spectra. In these numerical stud-

ies, various models of the scales of *Morpho*-butterfly were proposed, and succeeded in obtaining the reflection spectra which produce the *Morpho*-blue.

On the other hand, few numerical analyses with models of precise structure of a barb on structural color of birds' feather were reported except about the black-billed magpie (*Pica pica*) (Vigneron *et al.*, 2006), emphaLawes's Parotia (Wilts *et al.*, 2014), emphaScarlet macaw (Yin *et al.*, 2012) and the swallow tanager (*Tersina viridis*) (D'Ambrosio *et al.*, 2018; Skigin *et al.*, 2019). Although in a case of insects, Miyamoto and Kosaku found the micro-tubercles on the epidermis of the shieldbug responsible for the Tyndall blue (Miyamoto and Kosaku, 2002), in cases of birds, it is generally said to be vaguely due to the Tyndall phenomenon. It is mainly because the nano-scale structures of barbs of blue birds are too complex to specify the key structure that produces the structural-color. Hence, realistic modeling based on experiments and accurate simulations are required in order to understand the structural color of birds entirely.

In our previous work (Ueta *et al.*, 2014a), we proposed a simple porous flat plate model and the use of a highly reliable numerical method, that is, vector Korringa-Kohn-Rostoker (KKR) method (Ohtaka *et al.*, 1998) for high accuracy simulations beyond FDTD. We succeeded in explaining the experimentally obtained reflection spectrum to some extent qualitatively. However, although the measured reflectance has large value in the range of wavelength between 300 nm and 450 nm, that was not reproduced by the 2D simple porous slab model. That is, the obtained spectra do not reproduce the real color of the barb. In order to reproduce the real color, it should be necessary to model the whole barb, whereas the KKR is applicable only to the system which is made by periodically arranging spheres or cylinders, and also to such systems introduced some defects or disorders (D'Ambrosio *et al.*, 2018; Skigin *et al.*, 2019;

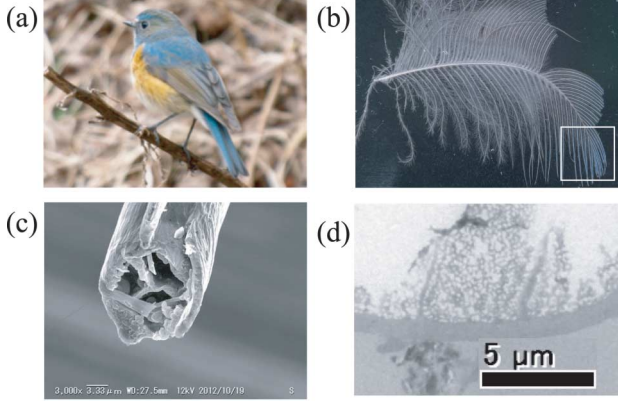


Fig. 1. *Tarsiger cyanurus* and their feather barbs. (a) Adult male of *Tarsiger cyanurus*. (b) A feather of *Tarsiger cyanurus*. (c) SEM image of the cross section of a barb. (d) TEM image of the porous structure of a barb composed of keratin (dark shadow and black) and air pore (light shadow).

Ueta *et al.*, 1998; Amemiya and Ohtaka, 2003; Urquia, *et al.*, 2020).

In the present study, we investigate optical properties of feather barbs of *Tarsiger cyanurus*, so called red-flanked bluetail, with more precise models taking account of the whole structure of a barb cross-section. The finite element method (FEM) (Ram-Mohan, 2002; Andonegui and Garcia-Adeva, 2013), which is applicable to problems of various fields throughout applied mathematics, engineering and physics, is employed.

The cross-sections of the barbs are observed by SEM and TEM, and the structural dimensions of the barbs are measured. The cross section of a barb is modeled by a two-dimensional (2D) ring with three radial partitions of equiangular interval. Both the ring and the partitions have a large number of pores. In order to reveal the origin of optical properties of the barbs, we propose four types of models of which arrangement and size of pores are different.

2. Modeling of a Feather Barb

2.1 Observation of barb-crosssection

Figures 1(a) and (b) show structural color of *Tarsiger cyanurus* and their barbs, respectively. Only adult males of *Tarsiger cyanurus* have blue colored feathers, whereas young males of less than about 2 years old and females have brown feathers. Barbs only at the tip of feather have blue color as shown in Fig. 1 (b) and a cross-sectional SEM image of the tip part of the blue barb is shown in Fig. 1(c). The cross-section of a barb has a structure which looks like three pointed star composed of a circular outer wall and three radial partitions. Figure 1(d) shows a TEM image of the outer wall in close up. The barb has a porous structure of keratin which includes spheroidal air-pores along the axis direction of the barb.

2.2 Modelling of barb-crosssection

By assuming continuous translational symmetry along the barb axis, we tackle the problem by means of 2D models as in our previous work (Ueta *et al.*, 2014a). In the previous work, the barb was modeled by a infinitely wide dielectrics slab with randomly arranged air rods in which the curvature

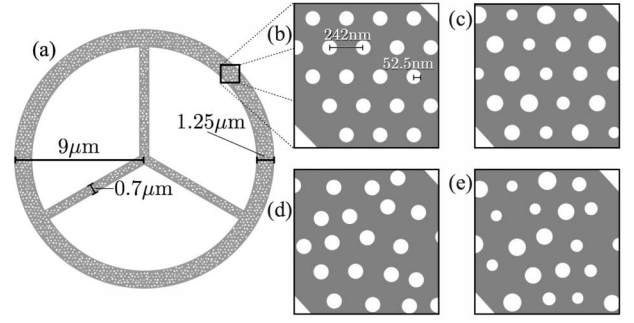


Fig. 2. Numerical models. (a) Cross-sectional structure of a barb composed of three radial partitions and the circular outer wall. (b) Periodic arrangement of the air pores of homogeneous radius. (c) Periodic arrangement of the air pores of random radius. (d) Random arrangement of the air pores of homogeneous radius. (e) Random arrangement of the air pores of random radius.

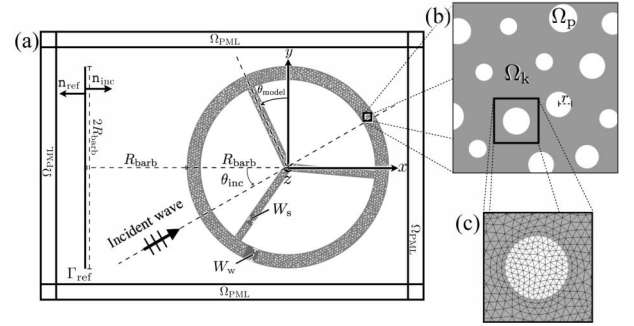


Fig. 3. Numerical computation scheme. (a) Whole scheme for reflectance computing. (b) Porous structure of the outer-wall in close-up. (c) Finite elements around a pore with radius 52.5 nm.

of the outer wall of a barb was neglected. In the present study, we define a precise 2D model beyond the previous model.

Numerical models of the blue-barb cross-section are shown in Fig. 2. The external radius of the outer-wall is $R_{\text{barb}} = 9.0 \mu\text{m}$, and thickness of the outer-wall and partitions are $1.25 \mu\text{m}$ and $0.7 \mu\text{m}$, respectively. Air-pores are distributed within both the outer-walls and partitions, and we propose four types of model with different arrangements and different sizes of air pores. Figure 2(b) shows homogeneous pores arranged in a triangular lattice. The randomness in the size of air-pores in Figs. 2(c), (e) and that in the arrangement in Figs. 2(d), (e) are determined by TEM observation of a number of barb samples.

The random radius a and the random position of $\mathbf{r}$ are defined by

$$a = \bar{a} + \sqrt{3}\sigma_a(2\xi_1 - 1), \quad (1)$$

$$\mathbf{r}_{m,n} = \mathbf{R}_{m,n} + \sqrt{\frac{3}{2}}\sigma_d(2\xi_2 - 1, 2\xi_3 - 1), \quad (2)$$

where $\bar{a}$ and $\mathbf{R}_{m,n}$ are the average radius of an air rod and the lattice vector of the triangular lattice given by

$$\mathbf{R}_{m,n} \equiv d(1, 0)m + d\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)n \quad (3)$$

with two independent integers m and n . d , σ_a and σ_d are the lattice constant of the triangular lattice, the standard deviation of the radius and that of the lattice constant, respectively. ξ_1 , ξ_2 and ξ_3 are independent uniformly distributed random numbers in the range 0 to 1. Then, the standard deviations of $\sqrt{3}\sigma_a(2\xi_1 - 1)$ and of the norm of the vector $\sqrt{\frac{3}{2}}\sigma_d(2\xi_2 - 1, 2\xi_3 - 1)$ becomes σ_a and σ_d , respectively.

On the basis of our measurements, the average radius of air rods is $\bar{a} = 52.5$ nm and the average interval between air-pores is $d = 242$ nm, the standard deviation of the radius is $\sigma_a = 7.5$ nm and the standard deviation of the lattice constant is $\sigma_d = 37$ nm. The scaled radius of the air-pore is equal to $a/d = 0.216942$.

3. Numerical Implementation by Finite Element Method

The numerical scheme of the system used to compute reflection spectra is illustrated in Fig. 3. We consider scattering of an incident plane wave toward the 2D model. The center of the circular outer wall is defined as the origin of x - y coordinate system, so that z -axis is toward the axis of the barb.

As shown in figures 6, 7, 9 of Ueta *et al.* (2014a), the reflection spectra of the TE and TM modes exhibited similar behavior in such a random porous system. In addition, we adopted the formulation with the electric field in order to avoid aggravating the numerical accuracy by the equation including the spacial derivative of the dielectric constant (Andonegui and Garcia-Adeva, 2013). The capacity of our workstation, however, prevents us from analyzing the TE mode with the electric field parallel to the z -axes which is described with a vector equation.

We, therefore, assume the electric field has only z component, that is TM polarization as $\mathbf{E} = (0, 0, E_z)^t$, where the superscript t denotes transposition of the vector. The incident angle θ_{inc} is defined as the angle between x -axis and the propagating direction of the incident wave. The rotation angle of the model is denoted by θ_{model} as shown in Fig. 3.

The governing equation of this problem is given by Maxwell's equations as follows:

$$\nabla \times \{\nabla \times \mathbf{E}_s\} - \frac{\omega^2}{c^2} \varepsilon(\mathbf{r}) \mathbf{E}_s = \frac{\omega^2}{c^2} [\varepsilon(\mathbf{r}) - \varepsilon_{\text{air}}] \mathbf{E}_i \quad (4)$$

where $\mathbf{r}$, $\mathbf{E}_i$ and $\mathbf{E}_s$ are the position vector, the incident and the scattered electric fields, respectively. The relative permittivity in air ε_{air} is set to unity. The quantity ω and c represent the angular frequency of electric field and the light velocity in a vacuum, respectively. The feather barbs are generally composed of keratin and air (Prum *et al.*, 1998). According to the previous work (Ueta *et al.*, 2014a), the spatial variation of the relative permittivity, $\varepsilon(\mathbf{r})$, is given as follows:

$$\varepsilon(\mathbf{r}) = \begin{cases} 2.43 & \mathbf{r} \in \Omega_k, \\ 1.00 & \mathbf{r} \in \Omega_{\text{air}}, \Omega_{\text{out}}, \Omega_p. \end{cases} \quad (5)$$

We simulate scattering of light by the numerical barb model in an open space by making use of PML absorbing boundary condition (Berenger, 1994) and of optimized absorbing function (Bermúdez *et al.*, 2007).

The number of nodes and elements are about 4×10^6 and 8×10^6 , respectively.

We consider the reflectance R on a reference line Γ_{ref} of length $2R_{\text{barb}}$ located at $x = -2R_{\text{barb}}$ as shown in Fig. 3. The reflectance R is defined as the ratio of the radiative power of scattered field to the incident one as follows:

$$R = \frac{\int_{\Gamma_{\text{ref}}} \text{Re} \left\{ \frac{\mathbf{E}_s \times \mathbf{H}_s^*}{2} \right\} \cdot \mathbf{n}_{\text{ref}} d\Gamma}{\int_{\Gamma_{\text{ref}}} \text{Re} \left\{ \frac{\mathbf{E}_i \times \mathbf{H}_i^*}{2} \right\} \cdot \mathbf{n}_{\text{inc}} d\Gamma} \Big|_{\theta_{\text{inc}}=0}, \quad (6)$$

where $\mathbf{n}_{\text{ref}}$ and $\mathbf{n}_{\text{inc}}$ are the outward normal unit vectors, respectively. Through the present paper we set $\theta_{\text{inc}} = 0$, so that the reflectance R becomes a function of θ_{model} . The integrand $\text{Re} \{ (\mathbf{E} \times \mathbf{H}^*)/2 \}$ represents the time average of Poynting vector. Here, the symbol “*” denotes the complex conjugate of the field.

We also define the sample-averaged reflectance $\langle R \rangle$. The average is performed over samples with different porous configurations.

4. Numerical Results

4.1 Influence of randomness of sizes and of arrangement of pores on the reflection spectrum

In order to measure the reflection spectra of blue feathers, we taped 10 feathers, which from the back of a male, on a black paper overlapping to recreate a blue color region of these feathers. The reflection spectra were measured by means of an Ocean Optics (Dunedin, FL, USA) USB2000 spectrometer and stored the data in a computer running OOIBASE32 software (range 300–700 nm). Reflected light at a right angle to the specimen was detected from a circular domain with a diameter of about 4 mm. The distance between the probe and specimen was set to 3 mm.

Figure 4(a) shows an experimentally measured reflection spectrum of a stack of 10 blue colored feathers. The value of reflectance becomes significantly large in the range of wavelength between 350 nm and 550 nm which corresponds to that of blue color. The profile of the reflection spectrum does scarcely depend on the samples and is very robust, so that the profile is considered to be produced by a definite origin. The noisy feature of the reflectance in the wavelength range beyond 700 nm is caused by setup of the spectrometer suitably adjusted to the wavelength range between 300 nm and 700 nm.

Figure 4(b) shows the numerically obtained reflection spectrum of the barb model without air-pores. The reflectance is small within the whole wavelength range. Several peaks are observed in the spectrum, whereas the typical high reflectance in the blue wavelength range is not present. Without pores, thus, it is not possible to explain the blue color.

We then show the reflection spectra computed for 4 kinds of model (Figs. 4(c)–(f)) in order to investigate influence of small air-pores on exhibiting the structural color.

The reflection spectrum of the model whose pores are homogeneous and placed periodically in a triangular lattice is shown in Fig. 4(c). We observe two major peaks around 360 nm and around 500 nm in the spectrum. The location of the peaks is obviously owing to the structure and the

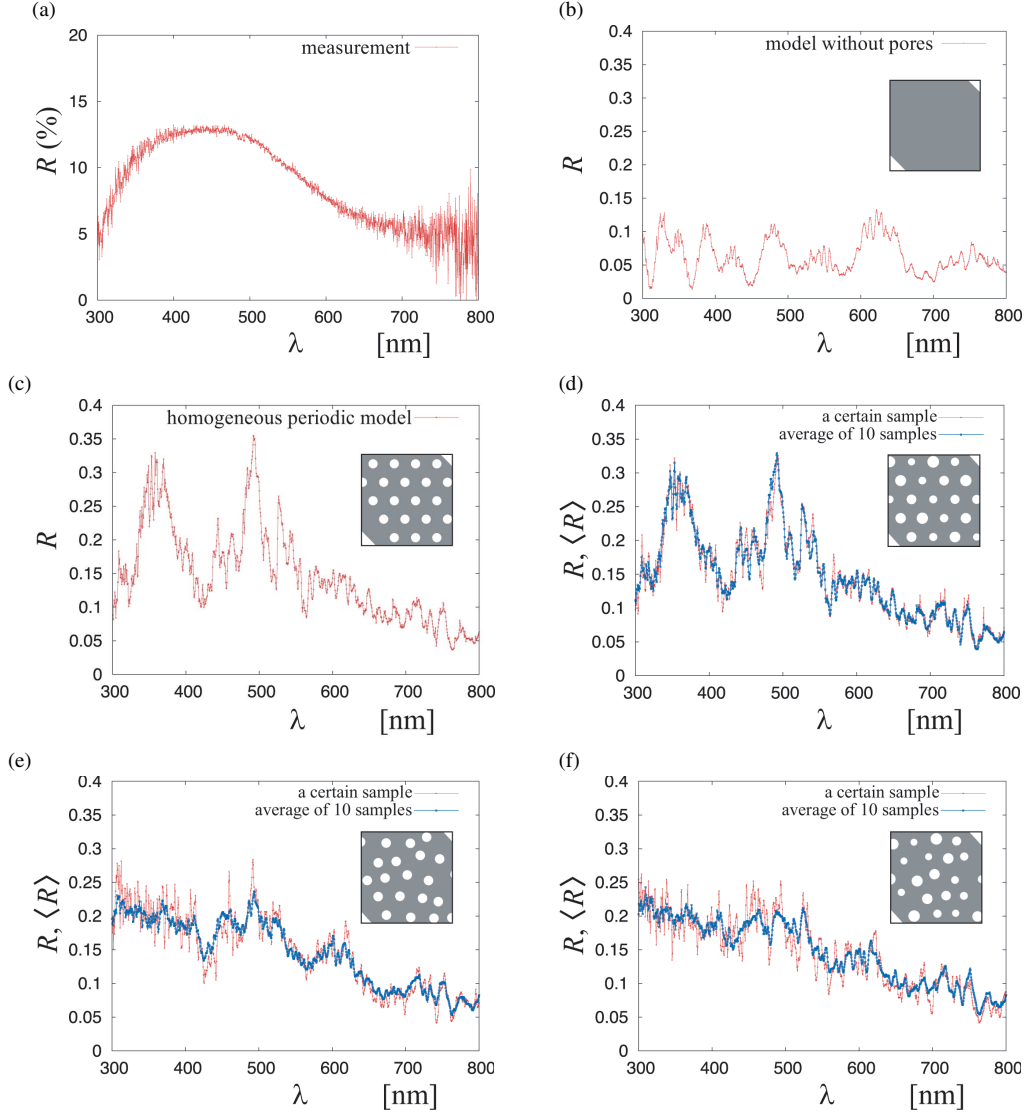


Fig. 4. Reflection spectra of the feather barb. (a) Experimental result. (b) The barb model without air-pores. (c) The barb model with periodically arranged air-pores of the homogeneous radius. (d) The barb model with periodically arranged air-pores of random radii. (e) The barb model with randomly arranged air-pores of the homogeneous radius. (f) The barb model with randomly arranged air-pores of random radii. The red line in (b)–(f) represents the reflection spectrum R of a certain sample and the blue one in (d)–(f) represents the averaged one $\langle R \rangle$ of 10 samples. The incident angle and the rotation angle are set to $\theta_{\text{inc}} = 0$, $\theta_{\text{model}} = 0$, respectively.

size of the lattice. Thus, Fig. 4(c) has two peaks in the range of 300–550 nm, so that it shows possibility that the porous structure within the outer-wall and the partitions plays a major role in reproducing the reflection spectrum in Fig. 4(a). Figures 4(d) shows the reflection spectra R and $\langle R \rangle$ of the case that the pores of random-radius are placed in the triangular lattice. The influence of the pores being still located in the triangular lattice appears in the reflection spectrum clearly. We see that the randomness of the radius of the pores scarcely affects the overall profile of the reflection spectrum. That is, it does not change significantly the color, still not blue. Both the spectra of Figs. 4(c) and (d) are converted to approximately an RGB triplet (137, 165, 163) of the 8-bit RGB color space (Ueta *et al.*, 2014b), and actually exhibit light teal gray. The color obviously differs from the structural color which we are considering here.

Figure 4(e) is the case of pores of homogeneous radius that are randomly distributed. When the position of the pores is disordered, both peaks around 370 nm and around 500 nm have washed out. We see that the influence of disorder in the position of the pores has strongly appeared in the short wavelength range of both reflection spectra.

The reflection spectra R and $\langle R \rangle$ of the model with both disorder of the position and radius of pores are shown in Fig. 4(f). Both the reflection spectra closely resemble that of Fig. 4(e), whereas we see that the vestiges of the peak around 500 nm have disappeared. The reflection spectra are overall decreasing linearly with respect to frequency, and the reflectance is larger for EM waves of shorter wavelength. It should look similar to Tyndall-blue produced by Rayleigh scattering but without the strong $1/\lambda^4$ dependence on the wavelength λ .

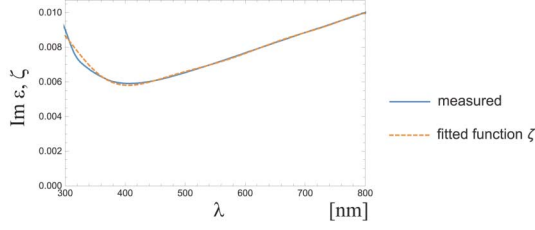


Fig. 5. Wavelength-dependence of the imaginary part of the dielectric constant of keratin (solid line) and the fitted function ζ (dashed line). The value of the imaginary part is scaled so as to be 0.01 at 800 nm.

4.2 Influence of absorption by keratin

Even the averaged reflection spectrum $\langle R \rangle$ in Fig. 4(f) seems not to agree with that of Fig. 4(a). Namely, the averaged reflection spectrum $\langle R \rangle$ in Fig. 4(f) is not attenuated in the ultraviolet range, whereas the reflection spectrum of Fig. 4(a) shows remarkable attenuation. That would be due to a certain physical origin. Attenuation of the reflectance in the ultraviolet range should be attributed to absorption by keratin, since electromagnetic waves are scattered more in the shorter-wavelength range.

In general, biopolymer materials absorb EM waves of short wavelength as, for example, the eye lens absorbs the ultraviolet radiation (Gaillard *et al.*, 2000). Therefore, now, we take into account the influence of the absorption of light by barbs.

In Japan, wild birds are strictly protected by law, so that it is almost impossible for us to prepare enough amount of specimen. Instead, we employ the profile of the absorption spectrum of barbs of a white leghorn, namely a white hen. The relative reflection spectrum was measured by JASCO Corporation and the imaginary part of the permittivity of keratin calculated from the data is plotted as a function of the wavelength λ [nm] in Fig. 5 with the fitted function ζ , that is, a power series of λ to order λ^{10} that best fits the data.

We defined the complex relative permittivity as

$$\varepsilon(\mathbf{r}) = \begin{cases} \varepsilon_k - \kappa^2 \zeta^2 / (4\varepsilon_k) + i\kappa\zeta & \mathbf{r} \in \Omega_k, \\ 1.00 & \mathbf{r} \in \Omega_{\text{air}}, \Omega_{\text{out}}, \Omega_p. \end{cases} \quad (7)$$

where $\varepsilon_k = 2.43$. Here, a scaling factor κ is introduced since the data of Fig. 5 pertain to the white leghorn than the red-flanked bluetail. The reflection spectrum computed with the complex relative permittivity is fitted to the measured one in terms of κ as a fitting parameter.

In Fig. 6, the averaged reflection spectrum $\langle R \rangle$ computed by using the complex relative permittivity Eq. (7) is shown for several values of κ . We see that the influence of the optical absorption on the reflectance becomes larger at shorter wavelength and especially the reflectance decreases remarkably in the range between 300 nm and 400 nm. The reflection spectrum for $\kappa = 1.0$ actually has a characteristic bump between 350 and 550 nm, and is quite similar to the measured one in Fig. 4(a).

4.3 Influence of the incident angle

Our numerical models have 3 radial partitions, so that in general the reflection spectrum is dependent on the rotation angle θ_{model} . In order to consider the influence of θ_{model} , we may define the rotation angle average $\overline{\langle R \rangle}$ of the sample-

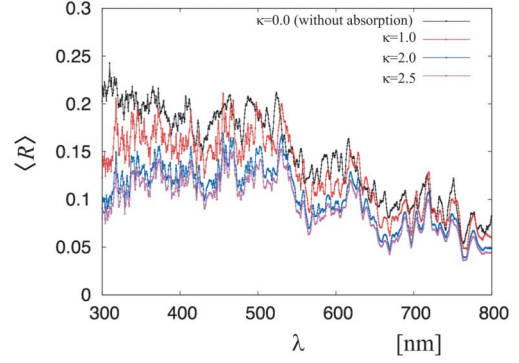


Fig. 6. The reflection spectra with absorption by keratin for several values of κ .

averaged reflectance $\langle R(\theta_{\text{model}}) \rangle$ as follows:

$$\overline{\langle R \rangle} = \frac{1}{2\pi} \int_0^{2\pi} \langle R(\theta_{\text{model}}) \rangle d\theta_{\text{model}}, \quad (8)$$

where the rotation symmetry of the system may be considered. However, scattering of light mainly occurs within the outer-wall and in the case of Fig. 4(f) the disorder is large enough, so that the major profile of the reflection spectrum is scarcely influenced by θ_{model} .

5. Conclusions

The cross-section of a barb of *Tarsiger cyanurus* has a structure of a cylindrical tube with three radial partitions. Within walls of the tube and the partitions, long spheroidal pores of random radius are randomly distributed. In the present study, we have proposed a more realistic 2D model of a barb of *Tarsiger cyanurus* than that of our previous work (Ueta *et al.*, 2014a).

Our model clearly exhibited that the granular air pores within the barbs scatter the incident light and reflect more strongly the light of shorter wavelength. In addition, we modeled the optical absorption spectrum of keratin by introducing the imaginary part to the permittivity based on the data of a white leghorn in order to explain the attenuation of the reflectance in the shorter wavelength range. It was found that optical absorption by keratin plays very important role in remarkably attenuating the reflectance in the wavelength range shorter than 400 nm. This situation is significantly similar to that of the wing of the morpho butterflies shown in figure 5 of Kinoshita *et al.* (2002b).

We can conclude that both effects are the origin of the profile of the measured reflectance of barbs of *Tarsiger cyanurus*. Further, the barb has three radial partitions within the cylindrical tube structure, so that we have considered the rotation angle of the system to the incident direction. The averaged reflection spectrum of 10 samples was averaged over the rotation angle. The major profile of the reflection spectrum is not affected very much by rotation angle averaging, namely the inner structure like the partitions.

The computed reflection spectrum, however, exhibits some oscillations with period dependent on the wavelength, though the measured reflectance does not show such a oscillation at all. Two causes for it strike us. One is that we computed the reflection spectra for a barb, whereas the ex-

perimental reflection spectrum was measured for a specimen of stacked 10 feathers including a lot of barbs. That is, the reflectance would be affected by scattering between feathers and between barbs. This might also be the reason that the numerical model in our previous study works well. The other is the influence of the non-uniformity of the axes direction of air-pores. We employed 2D model assuming that the air-pores are cylindrical and that its radius is constant in the axes direction, while actually the assumption would be broken. The analysis with the three-dimensional model taking account of the structure in the axis direction of this system is indeed very interesting. It is future work, whereas it is too difficult for us to perform it immediately since for that purpose we need to use a very large (super) computer system.

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