

Evaluation of Numerical Solution of Stochastic Differential Equations Describing Body Sway Using Translation Error

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Although many hypotheses, such as stiffness control and muscle mechano-reflex control, have been proposed for the elucidation of a control system to maintain an upright posture, the detailed mechanism is still unclear. In this study, the characteristics of muscle activity in the upright posture were investigated to verify the stiffness control hypothesis by employing stabilometry and the simultaneous measurement of electromyographs (EMGs). For each experimental test, with the subjects' eyes open/closed during stabilometry, EMGs were recorded for the gastrocnemius muscle, soleus muscle, and tibialis anterior muscles, simultaneously. We discuss a relationship between the center of gravity and muscle activity to control the upright posture. Mathematical models to control the standing posture are also discussed in this study.

Key words: Stabilometry, Body Sway, Temporally Averaged Potential Function (TAPF), Stochastic Differential Equation (SDE)

1. Introduction

Humans spend most of their everyday life in a standing posture. If it were difficult to maintain an upright posture, people would be significantly affected by cerebellar ataxia and other problems. Although many hypotheses have been proposed for the elucidation of a control system to maintain an upright posture, the detailed mechanism is still unclear. The standing posture in humans is maintained by the body's balance function, which is an involuntary physiological adjustment mechanism. This mechanism is referred to as the "righting reflex", which sensory receptors, such as proprioceptive inputs from the muscles, skin, and joints, auditory and vestibular functions, and visual inputs, are utilized to maintain the body's balance function [1].

Stabilometry is considered to be useful for the evaluation of the equilibrium function in a comprehensive way, and is expected to be employed not only in medical tests for otorhinolaryngology, but also in assessment tools for rehabilitation [2]. Measurement methods have been proposed to enhance the diagnostic value of stabilometry [3]. Stabilometry is typically performed with a subject standing in Romberg's posture. Romberg's posture is an upright posture with the feet placed together. It is an unstable standing posture, because the base of the support is narrow; thus, the body sway becomes marked, and a reduced equilibrium function is likely to be evident in stabilograms. Sways of the center of pressure on the grand plane are measured for 60 s at a time. The sway values, including the area of sway, total length, and locus length per unit area, are an analytical index for which the definition has been established by the Japanese Society for Equilibrium Research [4]. These

analytical indices have been used in many previous studies. The total length and the locus length per unit area are considered to represent microchanges in postural control, thereby serving as a scale of proprioceptive postural control. An objective evaluation is made possible through a computer analysis of the direction and speed of the sway, enabling a diagnosis of the patient's condition, such as tremors of Parkinson's disease or Meniere's disease [5].

Subjective deterioration of the equilibrium function was observed after the peripheral viewing of a 3D video clip in previous studies [6]. This persistent influence has been observed when the subjects view a poorly depicted background element peripherally, which generates a depth perception that contradicts daily life. Moreover, it has been mentioned that the sway values depend on the viewing period [7]. In an evaluation of the measured stabilograms, although a difference between the effect of a 3D video clip and a 2D video clip was observed during an eyes-closed examination, no difference was observed during an eyes-open examination [8]. Numerical solutions to mathematical models of body sway are known to be a countermeasure, which may contribute to the elucidation of a control system to maintain an upright posture. In this research, stabilograms taken when viewing a 3D video clip and a 2D video clip were evaluated by a numerical analysis [9].

2. Mathematical Models of Body Sway

In a stabilogram, variables y (the anterior direction, designated as positive) and x (the right direction, designated as positive) are regarded as independent [10]. To describe body sway, the linear stochastic differential equation (SDE) was proposed as a mathematical model (Brownian motion process) [11]–[13]. In particular, it is shown that it is necessary to extend the following nonlinear SDEs for a descrip-

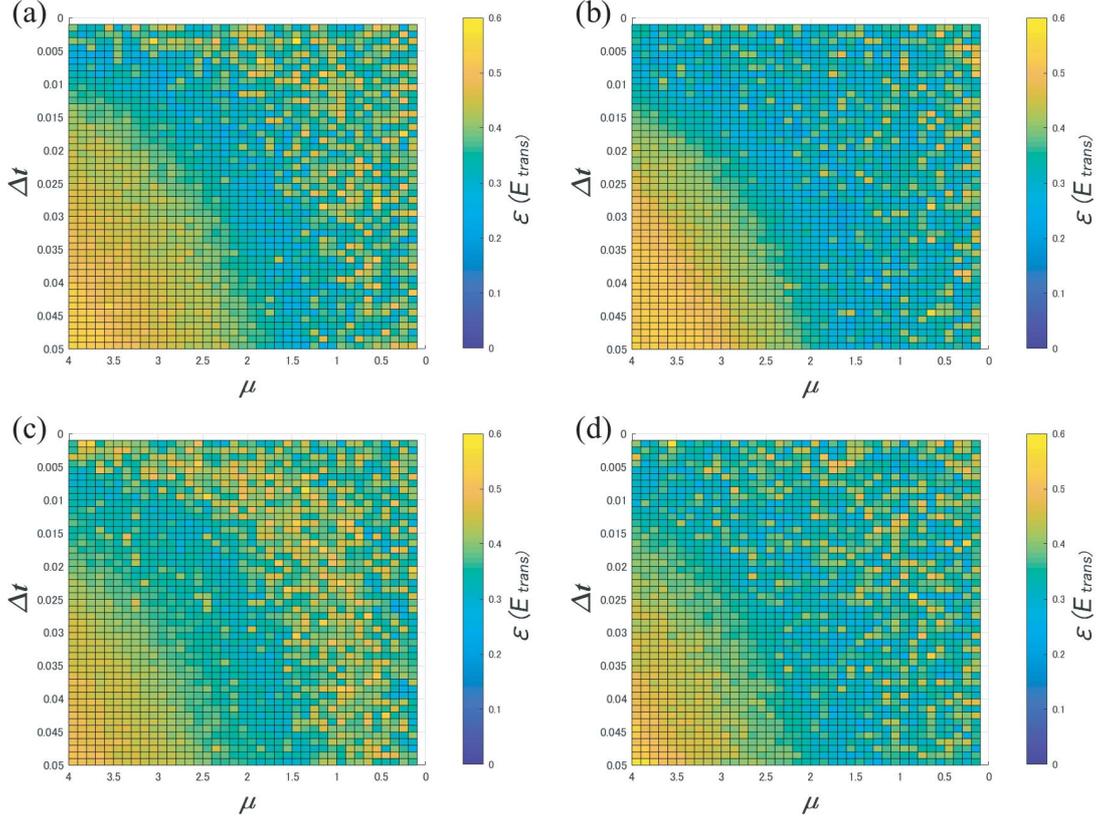


Fig. 1. Typical evaluation of numerical solutions in space spanned by the parameters μ and Δt in x -component for the young (a), in x -component for the elderly (b), in y -component for the young (c), and in y -component for the elderly (d).

tion of an individual body sway.

$$\frac{\partial x}{\partial t} = -\frac{\partial}{\partial x} U_x(x) + \mu_x \omega_x(t), \quad (1)$$

$$\frac{\partial y}{\partial t} = -\frac{\partial}{\partial y} U_y(x) + \mu_y \omega_y(t), \quad (2)$$

where μ_x and μ_y express the amplitudes of white Gaussian noises $\omega_x(t)$ and $\omega_y(t)$, respectively [14],[15]. In accordance with the previous theory, the noise amplitudes μ_x and μ_y were estimated by the numerical analysis for each component; otherwise, preprocessing for the standardization is required to compare multiple components in a polygraph. However, inconvenience is often experienced in the comparison of simultaneous measurements and/or biological loads. This problem has been eliminated in the present theoretical scheme [16]. The temporally averaged potential functions (TAPFs) $U_x(x)$ and $U_y(x)$ can be estimated by using the following formula, which is obtained from the stationary solution of the Fokker-Planck equation corresponding to SDEs (1) and (2) with the natural boundary conditions and constant fixation in the noise amplitudes. The correspondence can be shown by the calculation of the moment of transition probability.

$$U_x(x) = -\frac{\mu_x^2}{2} \ln G_x(x), \quad (3)$$

$$U_y(x) = -\frac{\mu_y^2}{2} \ln G_y(y), \quad (4)$$

where $G_x(x)$ and $G_y(y)$ represent the stationary distribution in each direction, whereas temporal variations of the distribution in each direction are expressed by the Fokker-Planck equation describing the Markov process without an abnormal dispersion.

Based on the observation, the stationary distribution can be estimated by the distribution measured for a sufficiently long time. In consideration of the nonlinearity of the biocontrol system, graphs of polynomials for degree four herein regress to the stationary distribution, as in Sec. 3. For perspective, the potential function can be estimated at each step when setting the width of moving windows for the mathematical analysis. Thus, the temporal variations of the potential function are described as a motion process in dual space [17]. Additionally, the SDEs can construct movements within the local stability with a high-frequency component near the minimal potential surface, where a high density is expected at the measurement point.

3. Statistical Processing and Numerical Analysis

Ten healthy elderly females (mean \pm standard deviation: 71.9 ± 4.09 years old) and ten healthy young females (21.1 ± 0.94 years old) voluntarily participated in this study. All were Japanese and lived in Nagoya and its surrounding areas. They provided informed consent prior to participation. The following subjects were excluded from the study: subjects working night shifts, those dependent on alcohol, those who consumed alcohol and caffeine-containing beverages after waking up and less than 2 h after meals,

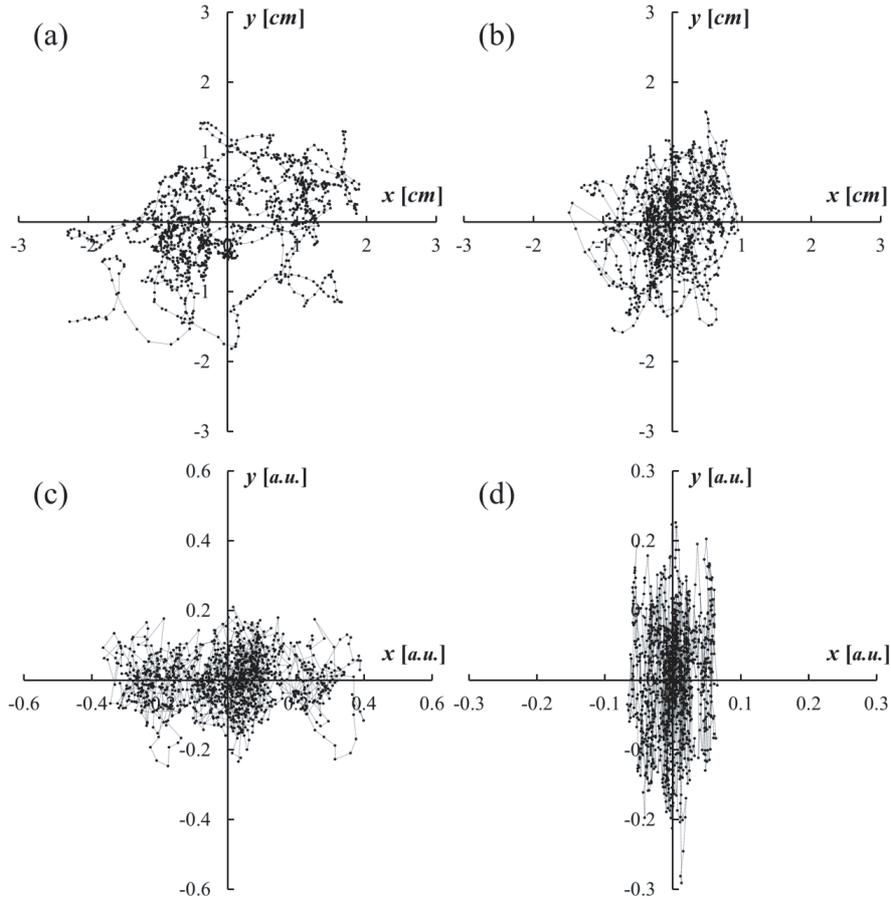


Fig. 2. Typical stabilograms: for a young female (a), for an elderly female (b), obtained from numerical solutions for the stabilogram (a) of a young subject (c), and obtained from numerical solutions for the stabilogram (b) of an elderly subject (d).

those using prescribed drugs, and those who may have had any otorhinolaryngologic or neurological disease in the past (except for conductive hearing impairment, which is commonly found in the elderly). This study was approved by the Research Ethics Committee of the Graduate School of Information Science, Nagoya University.

Stabilometry was conducted on subjects gazing at a visual target 2 m ahead. They stood on the stabilometer with a Romberg posture. The motion process of the body sway for each component is assumed not to be with anomalous diffusion but to be generated by Markov process. Also, components of the body sway are assumed to be independent [10]. A SDE was obtained from the time series data measured in this stabilometry as a mathematical model of the body sway for each component.

Through statistical processing, the mean of each stabilogram was first set to the original point O. Then, a histogram in each direction was obtained from a stabilogram. Next, a TAPF was estimated from a histogram by using the following polynomials: shown that it is necessary to extend the following nonlinear SDEs for a description of an individual body sway.

$$\hat{U}_x(x) = a_x x^4 + b_x x^3 + c_x x^2 + d_x x + \text{const.}, \quad (5)$$

$$\hat{U}_y(y) = a_y y^4 + b_y y^3 + c_y y^2 + d_y y + \text{const.} \quad (6)$$

The TAPFs in each direction for each group were regressed

through polynomials of degree four. Weak nonlinearity could be observed in the potential function, especially in the lateral direction of the young subjects. In general, the variance of a stabilogram depends on the TAPF with those several minimum values. Substituting Eqs. (5) and (6) into Eqs. (1) and (2), the SDEs

$$\frac{\partial x}{\partial t} = -\frac{\partial \hat{U}_x(x)}{\partial x} + \mu_x \omega_x(t), \quad (7)$$

$$\frac{\partial y}{\partial t} = -\frac{\partial \hat{U}_y(y)}{\partial y} + \mu_y \omega_y(t), \quad (8)$$

were finally determined as mathematical models of the body sway. Numerical solutions were obtained using the Runge-Kutta formula as the numerical calculus. The initial values of (x, y) were set to the original point O. In Eqs. (7) and (8), $\omega_x(t)$ and $\omega_y(t)$ are pseudorandom numbers (with mean \pm standard deviation of 0 ± 1.0) produced by white Gaussian noise. The time step Δt was set for every 0.001 step, from 0.001 to 0.05. The noise amplitudes μ_x and μ_y were set for every 0.1 step, from 0.1 to 4. A numerical analysis was applied for 11,200 steps, and the first 10,000 steps were discarded owing to the dependence of the initial value.

4. Evaluation Methods of Numerical Solution

In previous research, numerical solutions were evaluated using the following method [16].

Table 1. Optimal parameters in the numerical simulations: in the simulative stabilogram (Fig. 2c) of a young subject and in the simulative stabilogram (Fig. 2d) of an elderly subject.

In the simulative stabilogram of a young subject		
	x	y
Δt^*	0.040	0.010
μ	0.6	4.0
$\mu\Delta t^*$	0.024	0.040
In the simulative stabilogram of an elderly subject		
	x	y
Δt^*	0.006	0.020
μ	0.4	1.9
$\mu\Delta t^*$	0.0024	0.038

1. The total locus length Ls and area of sway Ss were calculated in the numerical solutions.
2. Denoting the measured total locus length and area of sway by Lr and Sr , the error ϵ between the numerical solutions of the mathematical model and measured data was defined as

$$\epsilon = \sqrt{\frac{\sqrt{Sr}}{Lr}(Lr - Ls)^2 + (\sqrt{Sr} - \sqrt{Ss})^2}. \quad (9)$$

3. Optimal values for parameters μ and Δt were estimated as follows. At $\Delta t = 0.001, 0.002, \dots, 0.05$, the error ϵ was first estimated every 0.01 step for $0 < \mu < 4$. Second, an optimal value of the parameter μ was estimated every 0.001 step for $0.001 < \Delta t < 0.05$, which resulted in the smallest error ϵ . This smallest value of ϵ was determined to be $\epsilon(\Delta t)$. Third, the variance of ϵ on the $\epsilon - \Delta t$ plane, namely,

$$V_{\Delta t^*}[\epsilon] = \frac{1}{N} \sum_{\Delta t^* \leq \Delta t \leq 0.05} \{\epsilon(\Delta t) - E[\epsilon]\}^2 \quad (10)$$

was calculated to determine the plateau of E , where N represented the number of samples $\epsilon(\Delta t)$ for $\Delta t^* \leq \Delta t \leq 0.05$, i.e. $N = (0.05 - \Delta t^*)/0.001$, and $E[\epsilon]$ expressed the expectation value for this interval of the time step. Finally, the optimal value of Δt^* was estimated when the variance exceeded 0.3 for the first period.

However, there was a problem when using the above-mentioned evaluation method as the scale of the numerical solution was lower than the extent of the actual stabilograms. For this reason, a considerable value of $\mu\Delta t$ with a high coincidence ratio between the actual stabilogram and the total locus length was chosen as the optimal value. In this study, the scale of the numerical solutions was corrected by multiplying it by the expansion scale Lr/Ls .

Based on the total locus length for the expansion scale, the total locus length of the numerical solution had the same value as the actual stabilogram. Here, the error ϵ does not depend on the total locus length, but on the area of sway, only when Eq. (9) is used for the evaluation of the numerical solution. In particular, one cannot take the information of the fluctuation in the body sway into consideration.

Thus, it is believed that ϵ in Eq. (9) is inappropriate to verify whether the numerical solution fits the time series data in each direction of the stabilograms used in previous studies [16]. Therefore, in this study, the focus was on the following indices of the numerical solution: the translation errors estimated from the time series data of the body sway for each component and the sequences of their temporal differences for each subject. The size of the errors from the actual value was calculated via the following equation:

$$\epsilon(E_{trans}) = \sqrt{\left\{E_{trans}^{(r)} - E_{trans}^{(s)}\right\}^2 + \left\{E'_{trans}{}^{(r)} - E'_{trans}{}^{(s)}\right\}^2} \quad (11)$$

The translation errors estimated based on the Wayland algorithm $E_{trans}^{(s)}$ and those estimated based on the Double Wayland algorithm $E'_{trans}{}^{(s)}$ were also calculated in the numerical solutions. The measured translation errors were denoted by $E_{trans}^{(r)}$ and $E'_{trans}{}^{(r)}$, and the error $\epsilon(E_{trans})$ between the numerical solutions of the mathematical model and measured data was defined (Fig. 1). The combination of μ and Δt with the smallest error was determined as the optimal value. One can identify multiple valleys, especially in the results of the young subjects (Figs. 1a and 1c), suggesting that there are fractal structures in the time step of the control system to maintain an upright posture. Henceforth, the combination of μ and Δt for the optimal value is expressed as $\mu\Delta t^*$ (Table 1).

The stabilograms of the elderly (Figs. 2b and 2d) were compared with those of the young (Figs. 2a and 2c). Figures 2c and 2d depict the simulated stabilograms obtained from a young and an elderly person. It should be noted that there were remarkable differences between suitable values of the simulative parameters in the youth and those in the elderly (see Table 1). Extreme anisotropy was seen in the graph of numerical solutions for the elderly, however, degree of rigidity seems to be enhanced with aging in the lateral component.

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